Advancing axion physics with AMReX

Malte Buschmann
DESY

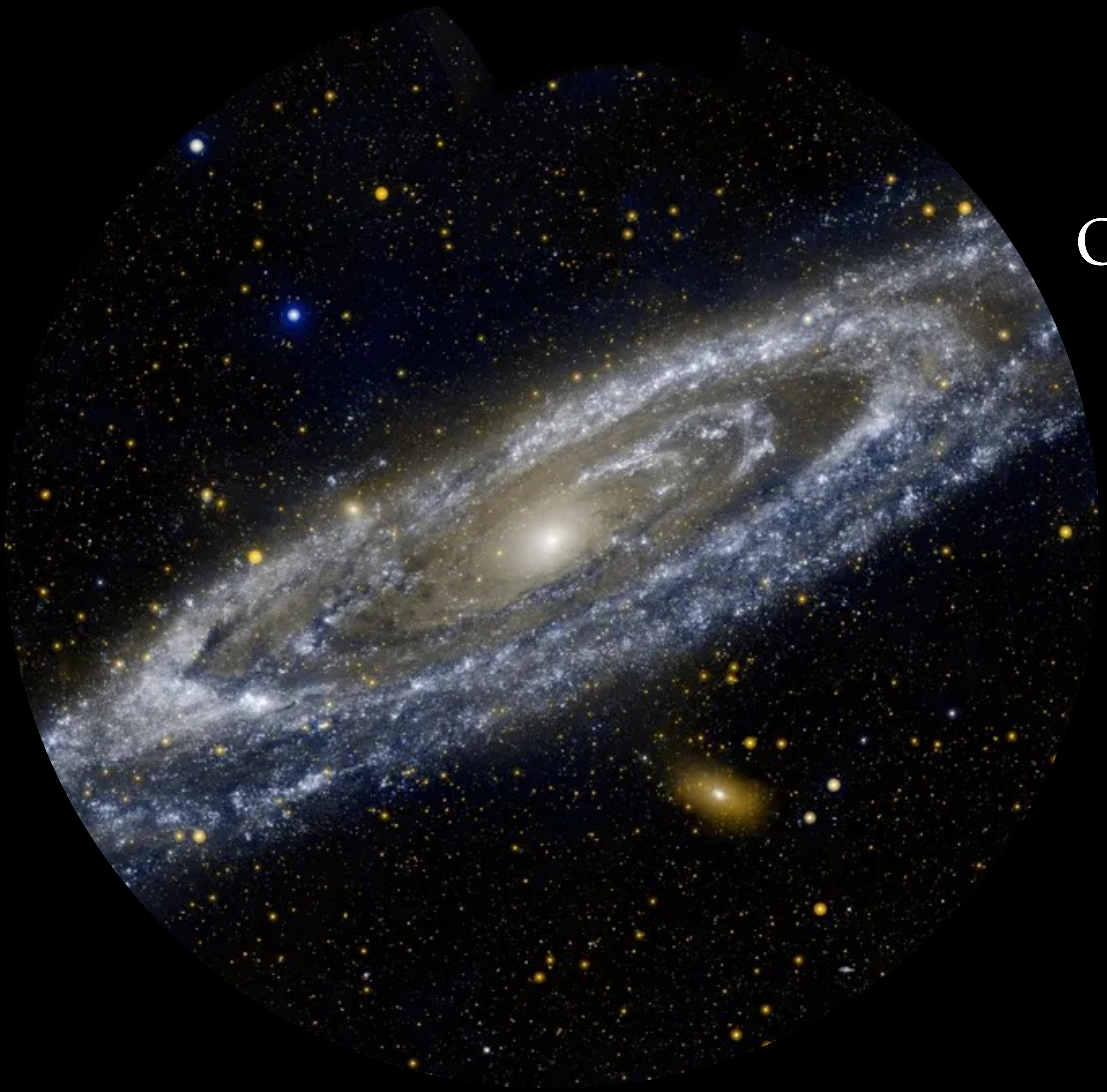
Mostly based on:

Phys.Rev.Lett. 134 (2025) 24, 241003 • e-Print: 2412.08699

Nature Commun. 13 (2022) 1, 1049 • e-Print: 2108.05368

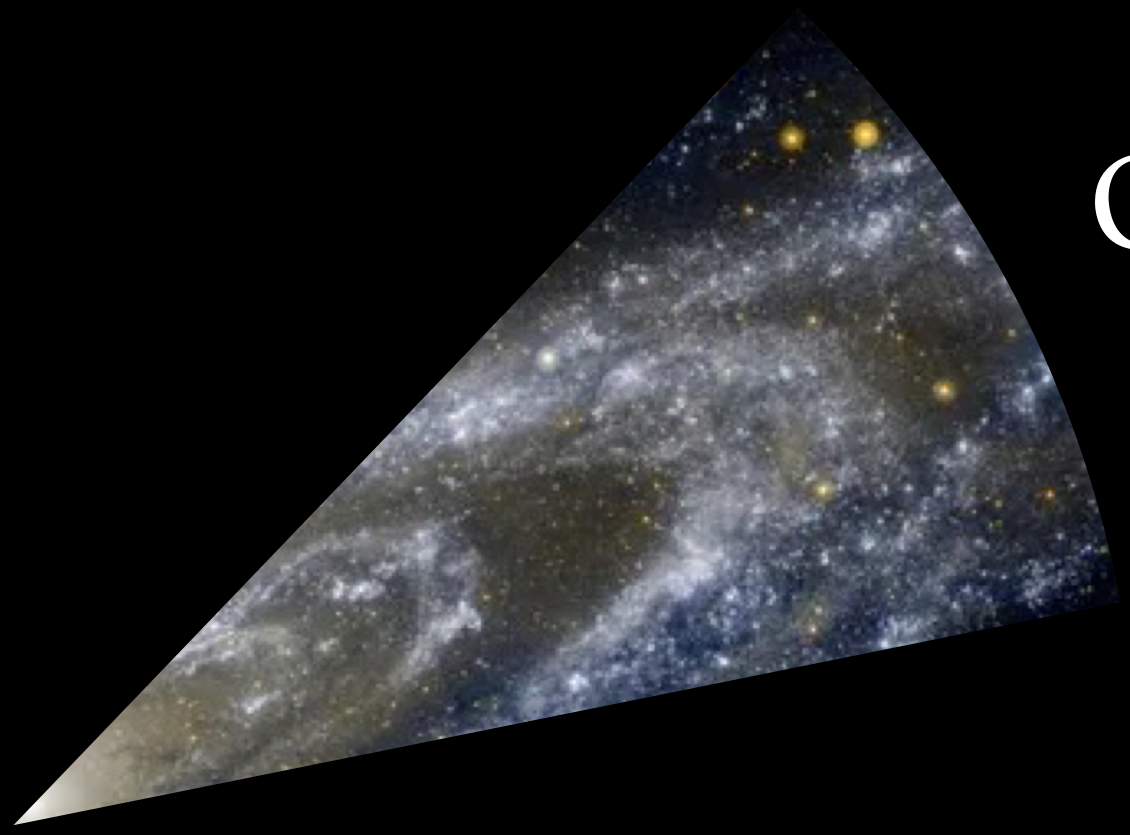
Phys.Rev.Lett. 124 (2020) 16, 161103 • e-Print: 1906.00967

THE MATTER CONTENT OF THE UNIVERSE



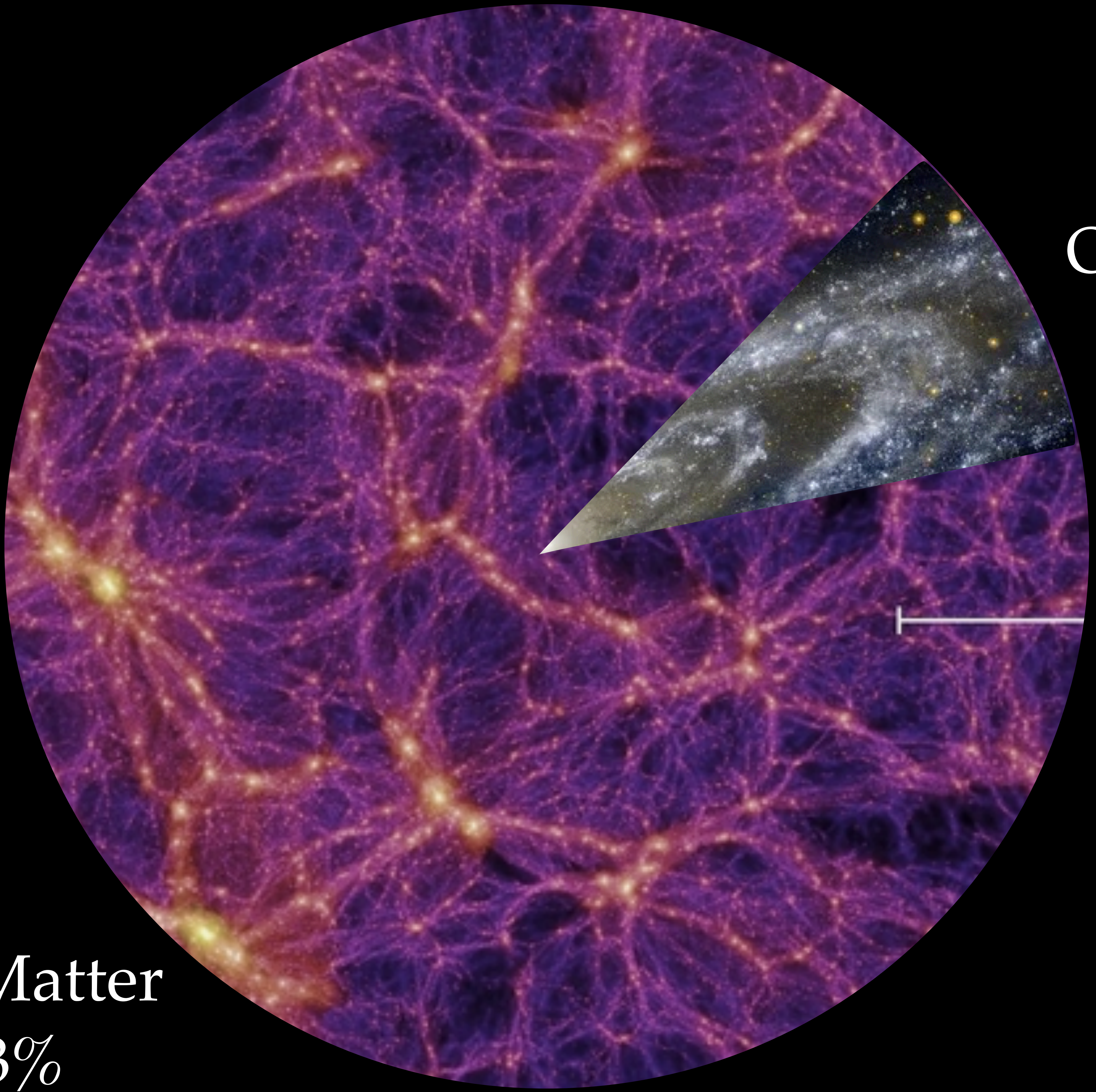
Ordinary Matter

THE MATTER CONTENT OF THE UNIVERSE



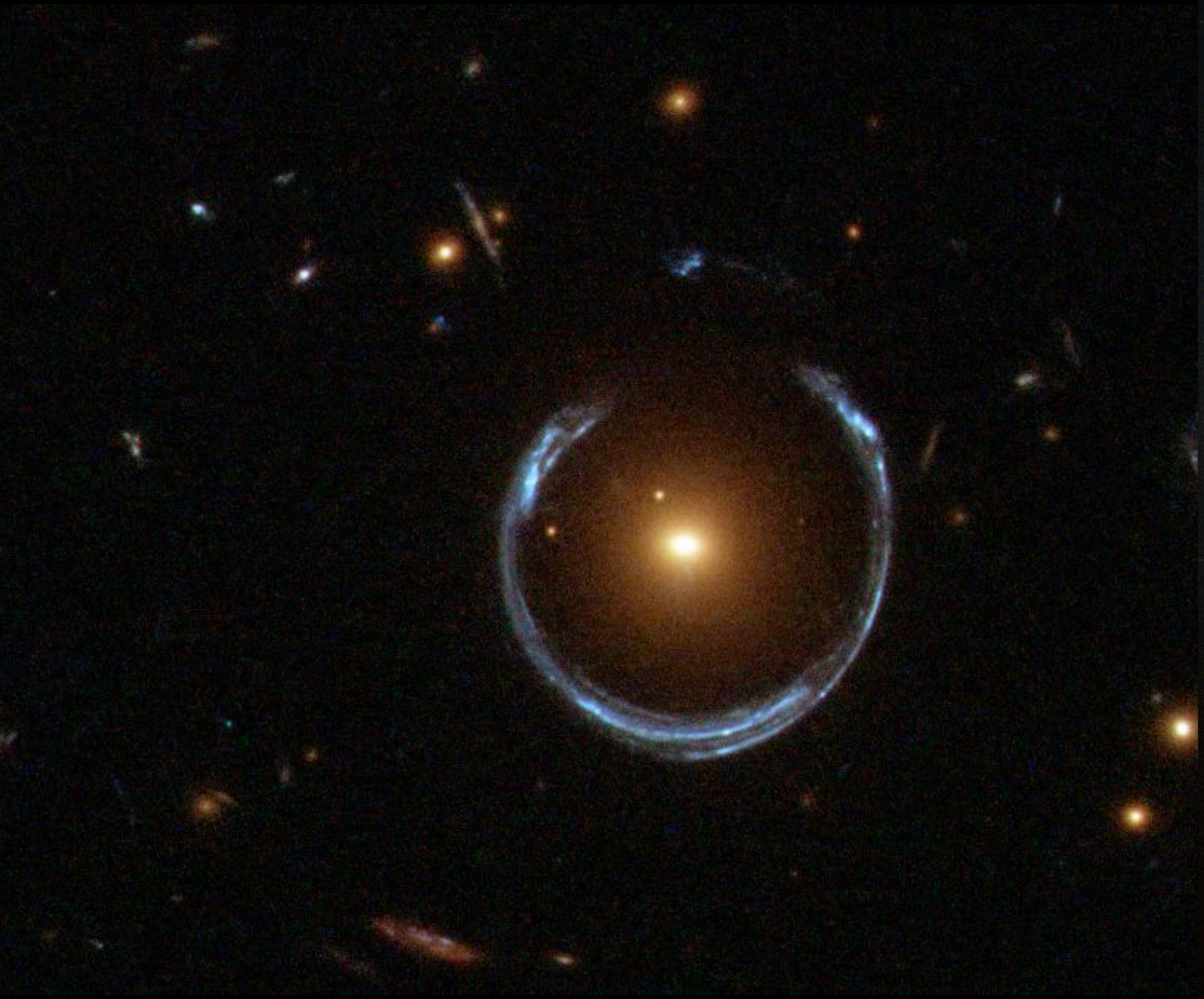
Ordinary Matter
~17%

THE MATTER CONTENT OF THE UNIVERSE

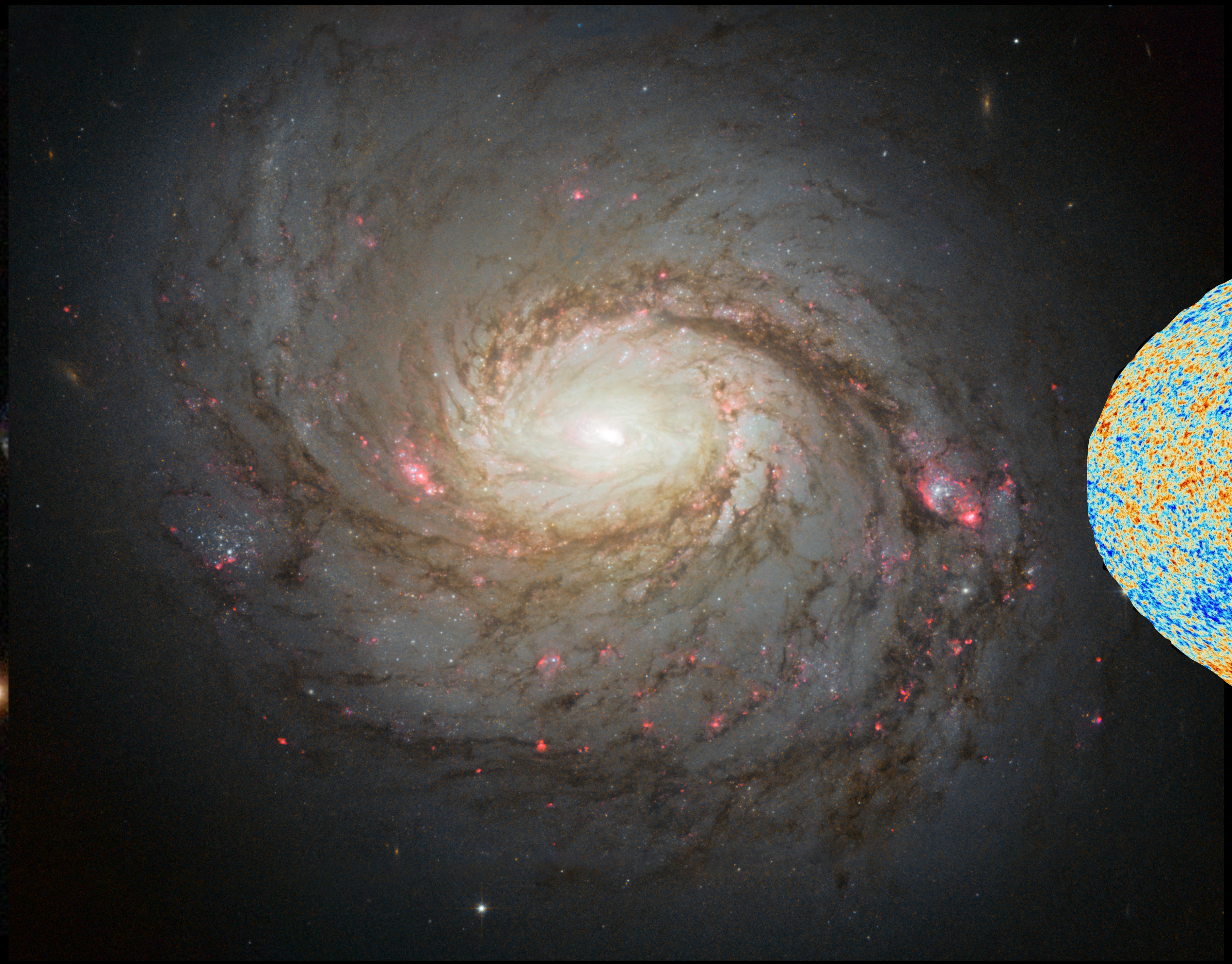


Dark Matter: We know it exists, but not what it is

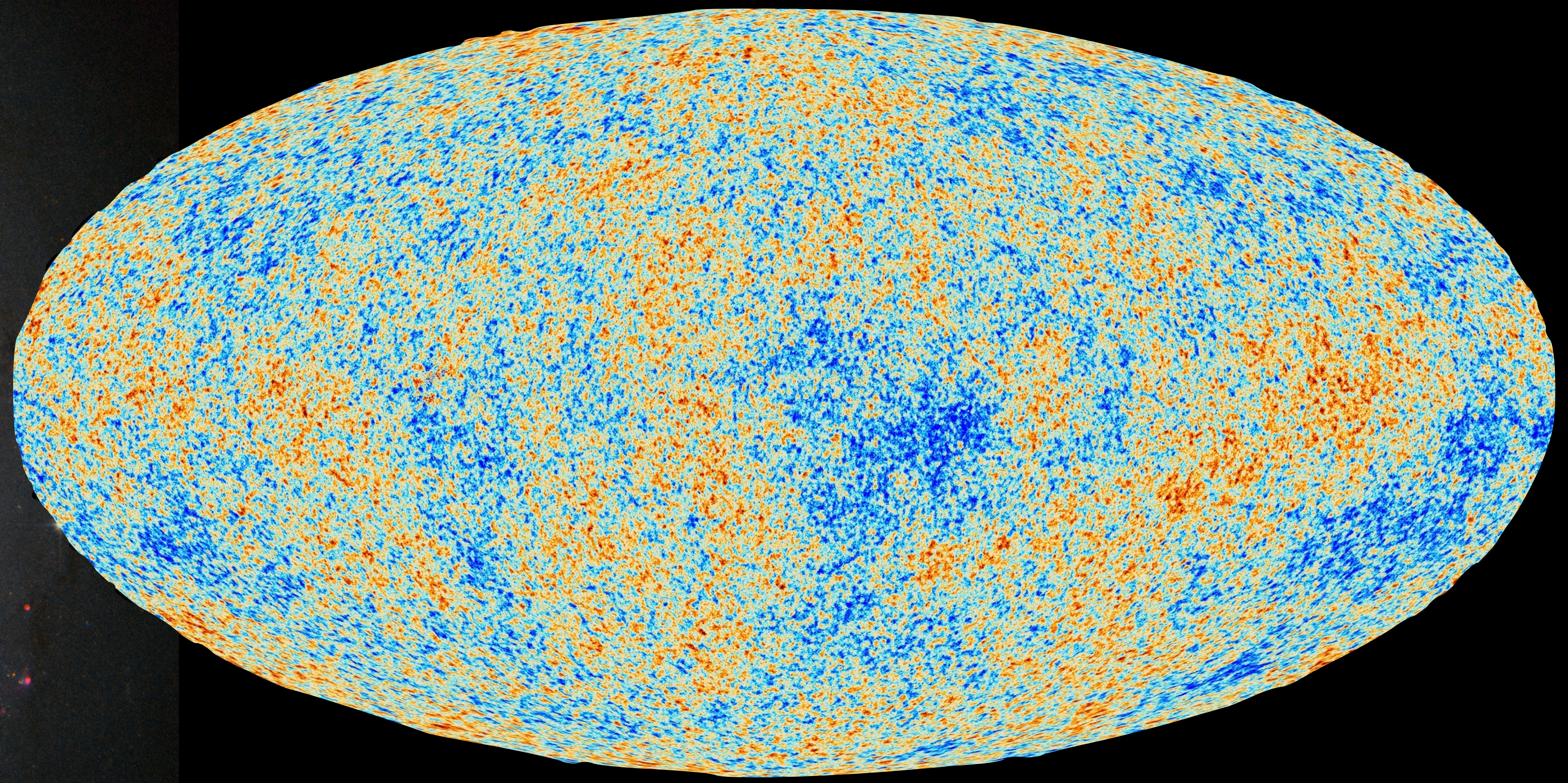
Gravitational Lense



Galaxy Formation



Cosmic Microwave Background



We can observe its gravitational pull

Dark Matter: We know it exists, but not what it is

Theories:

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SUSY

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SUSY (SUper SYmmetry)

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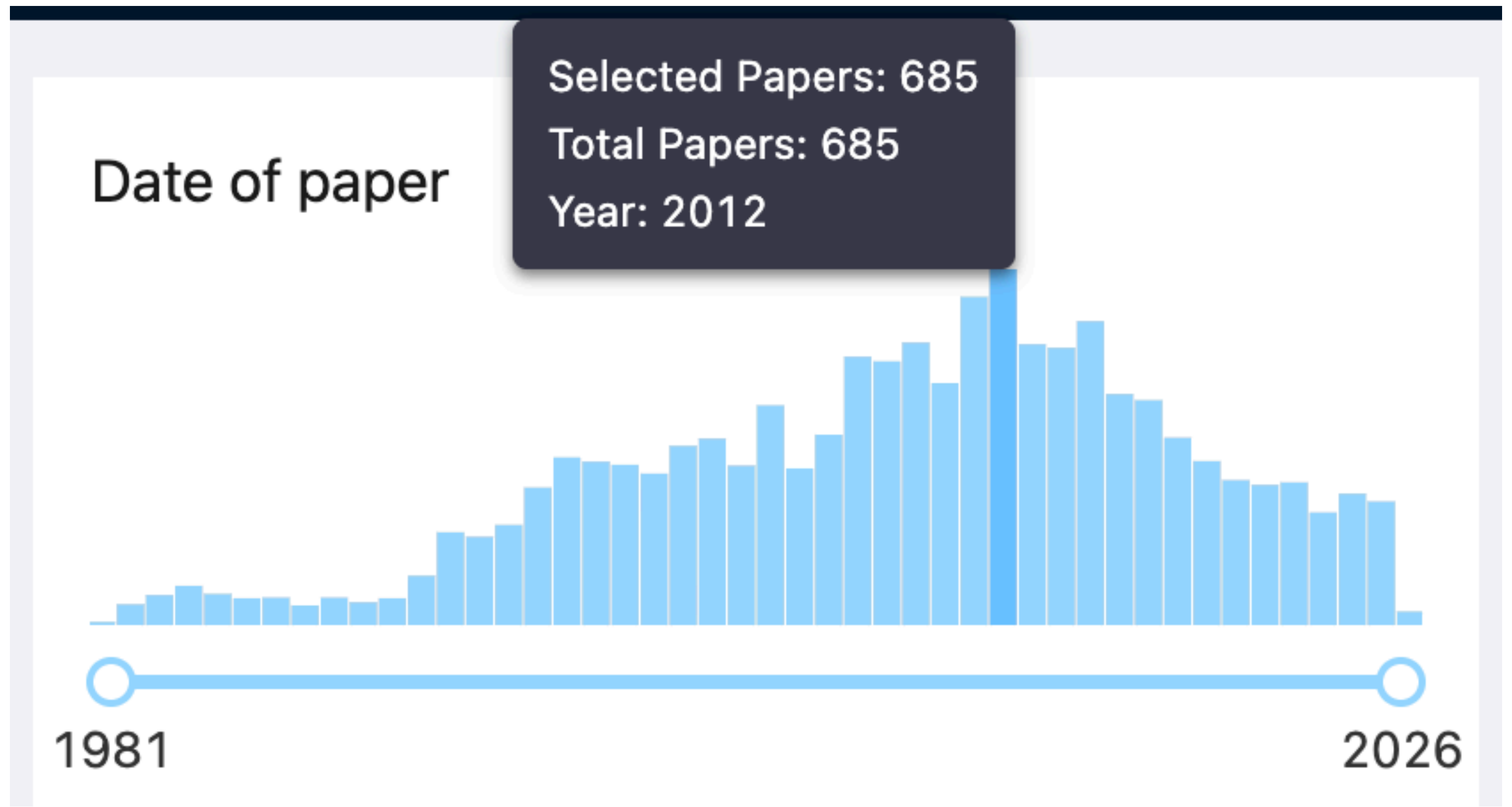

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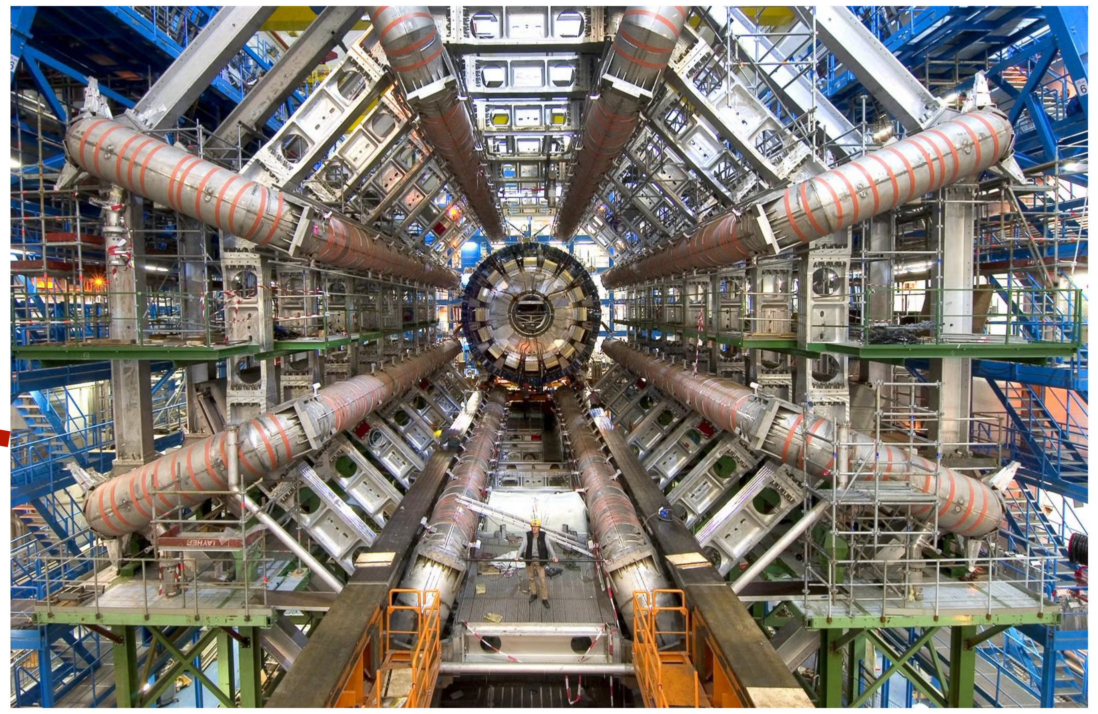

First data @ LHC at CERN

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WIMPs

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Theories:

SUSY	**(SUper SYmmetry)**
WIMPs	**(Weakly Interacting Massive Particle)**

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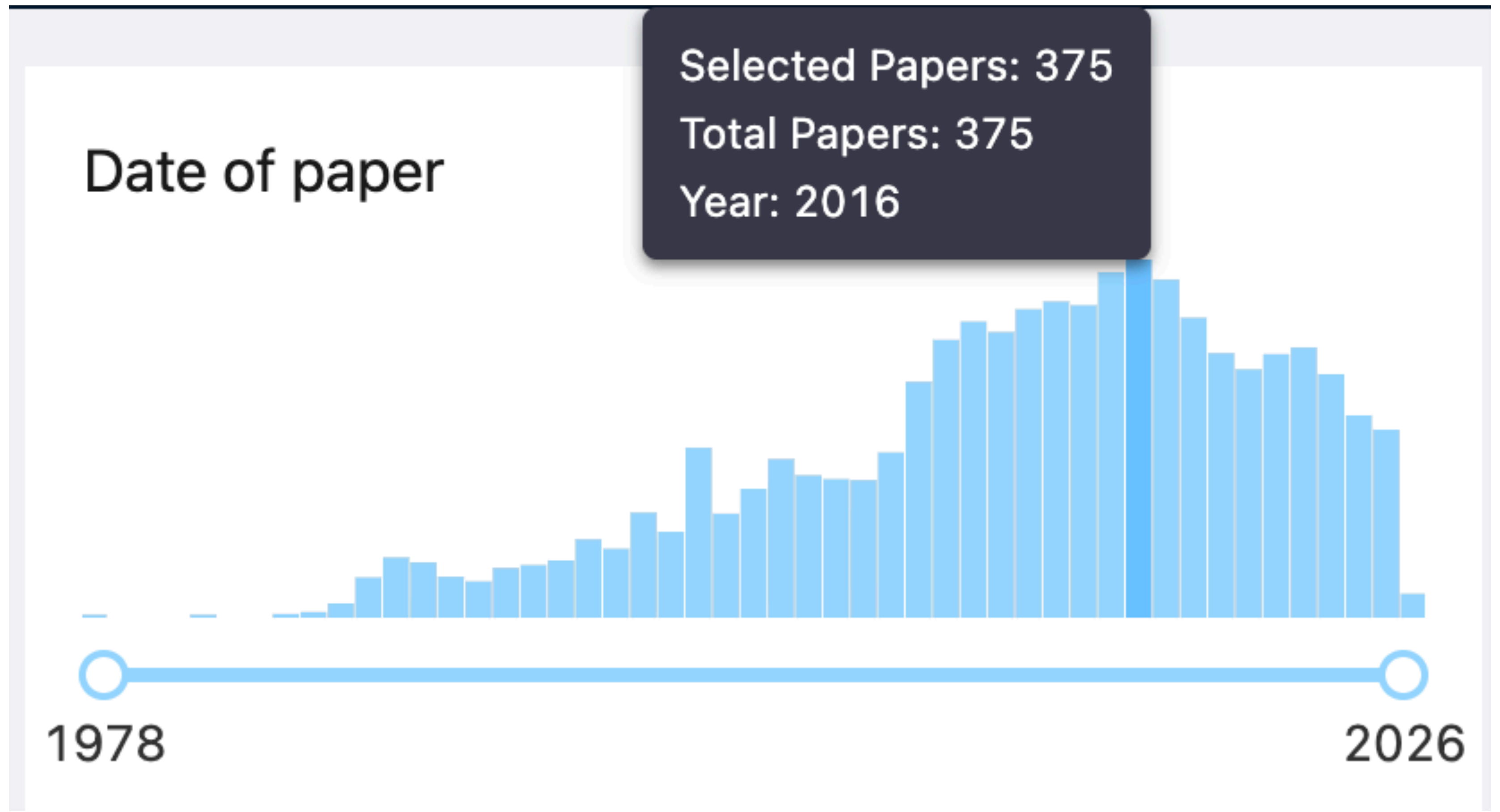

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First data of LUX experiment

Dark Matter: We know it exists, but not what it is

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MACHOs

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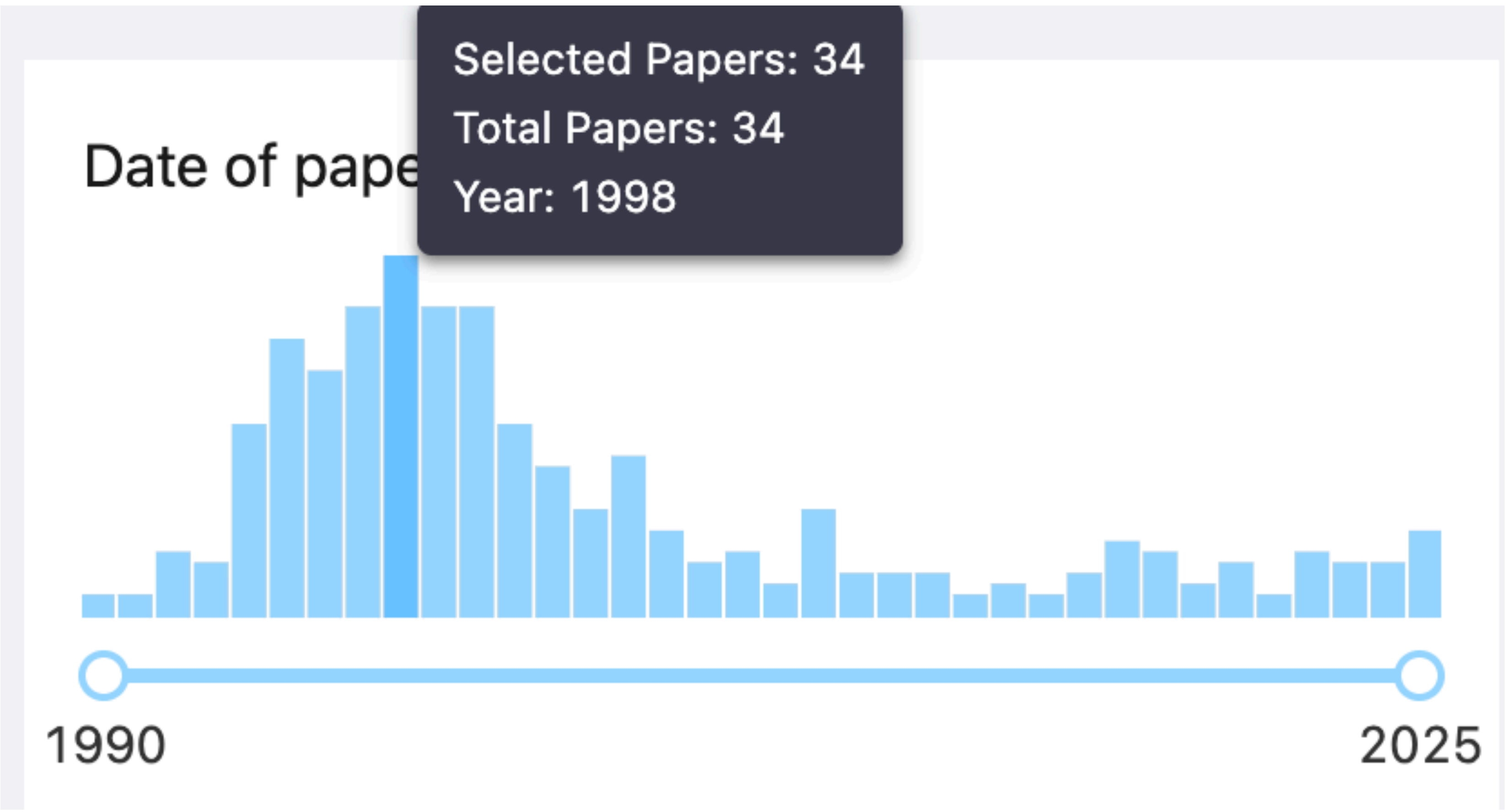

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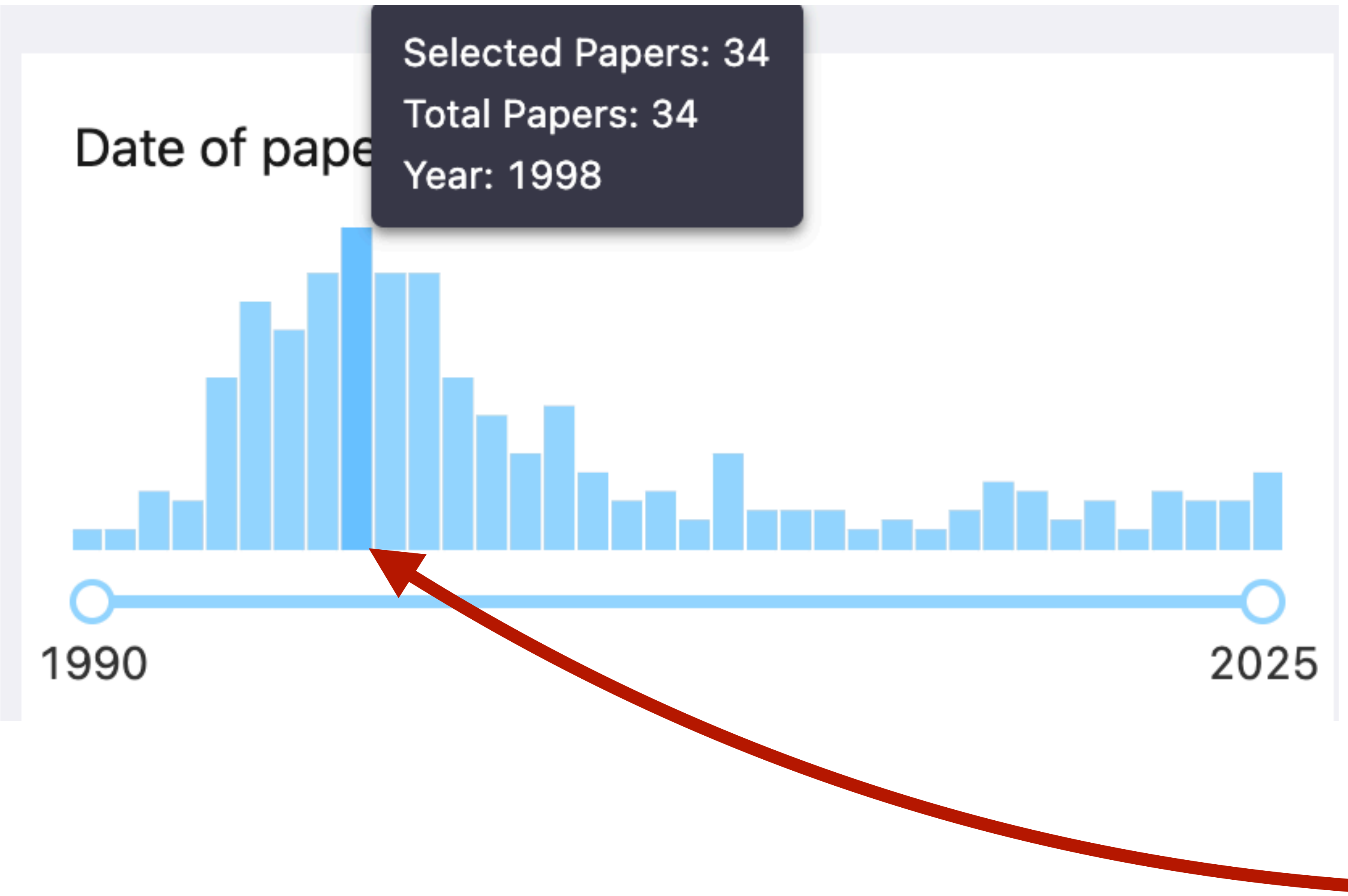

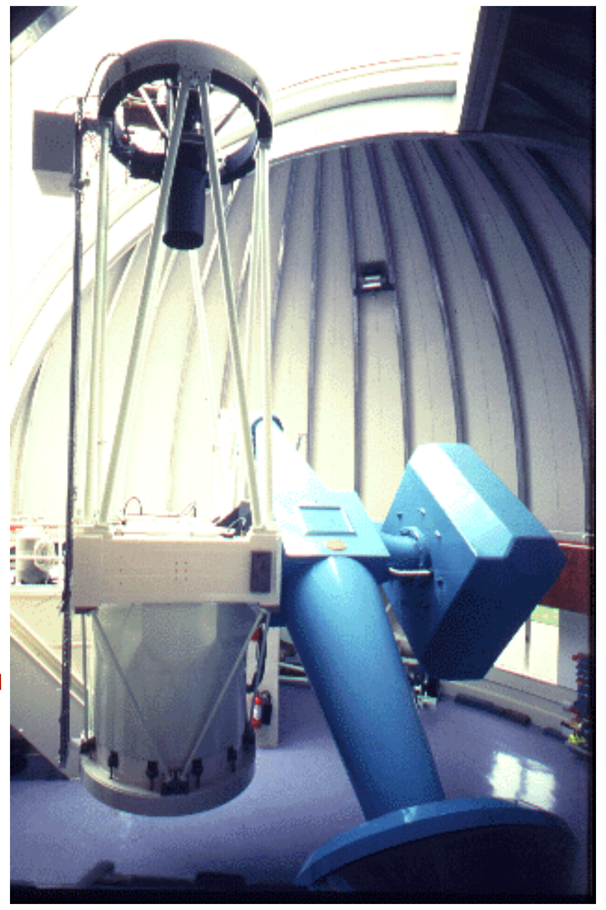

First data of MACHO experiment

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Axions	

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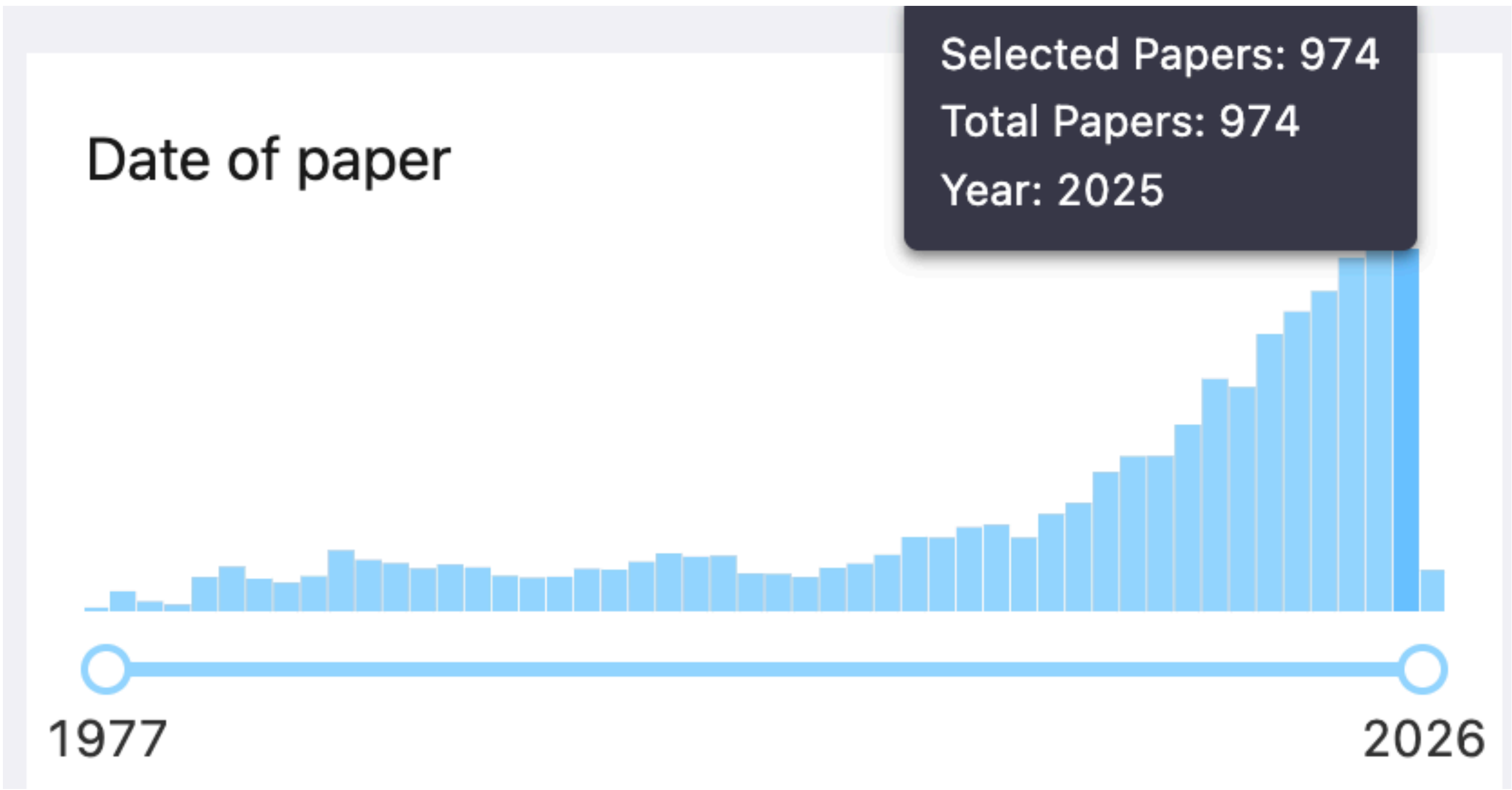

Very much alive!

One promising candidate: The "Axion"

A new elementary particle that can explain dark matter
but also the strong CP problem

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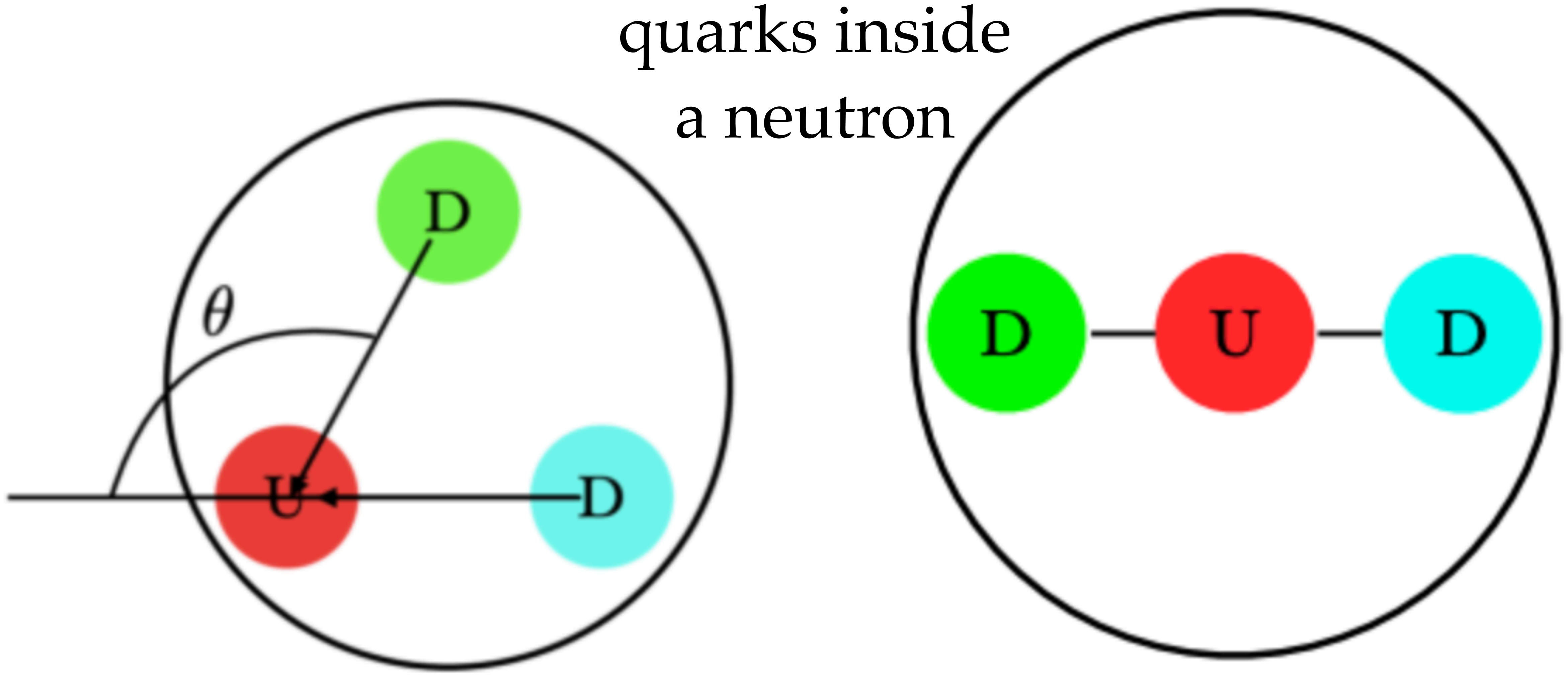
Arrangement of
quarks inside
a neutron



Predicted

Measured

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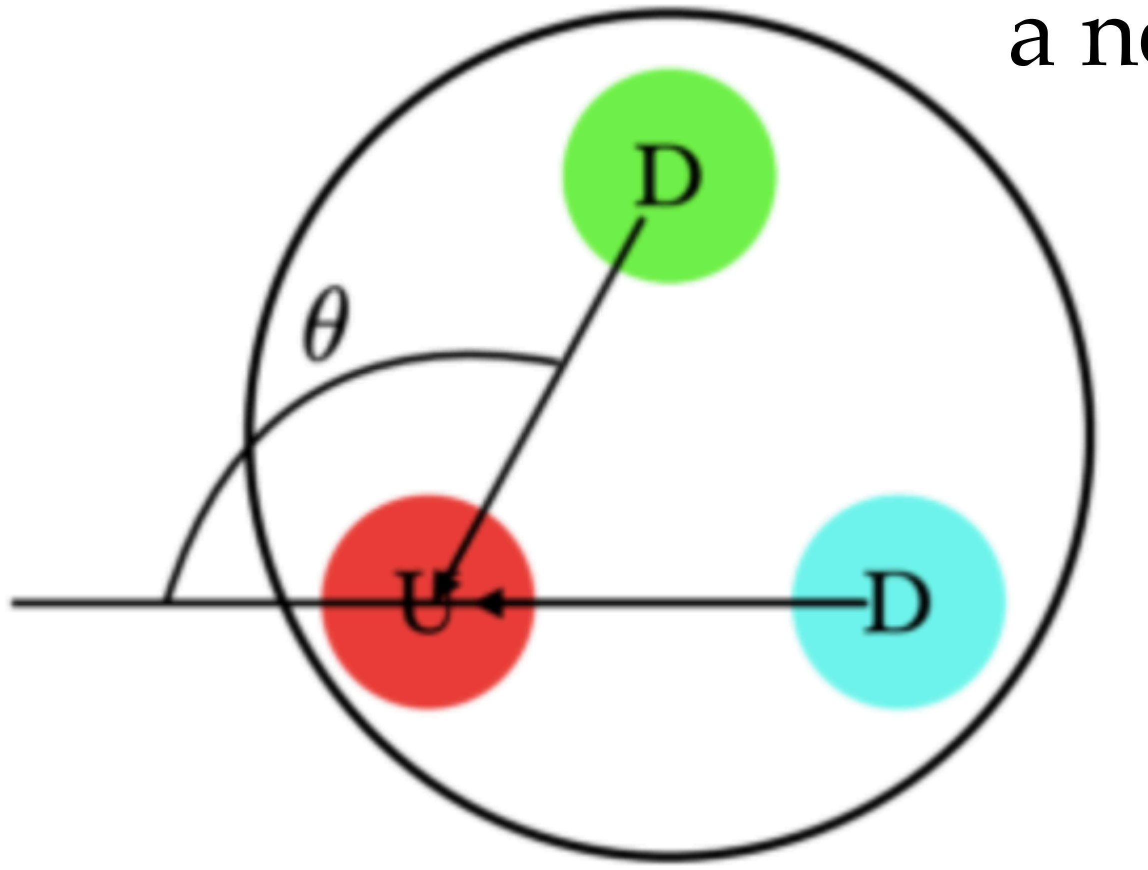

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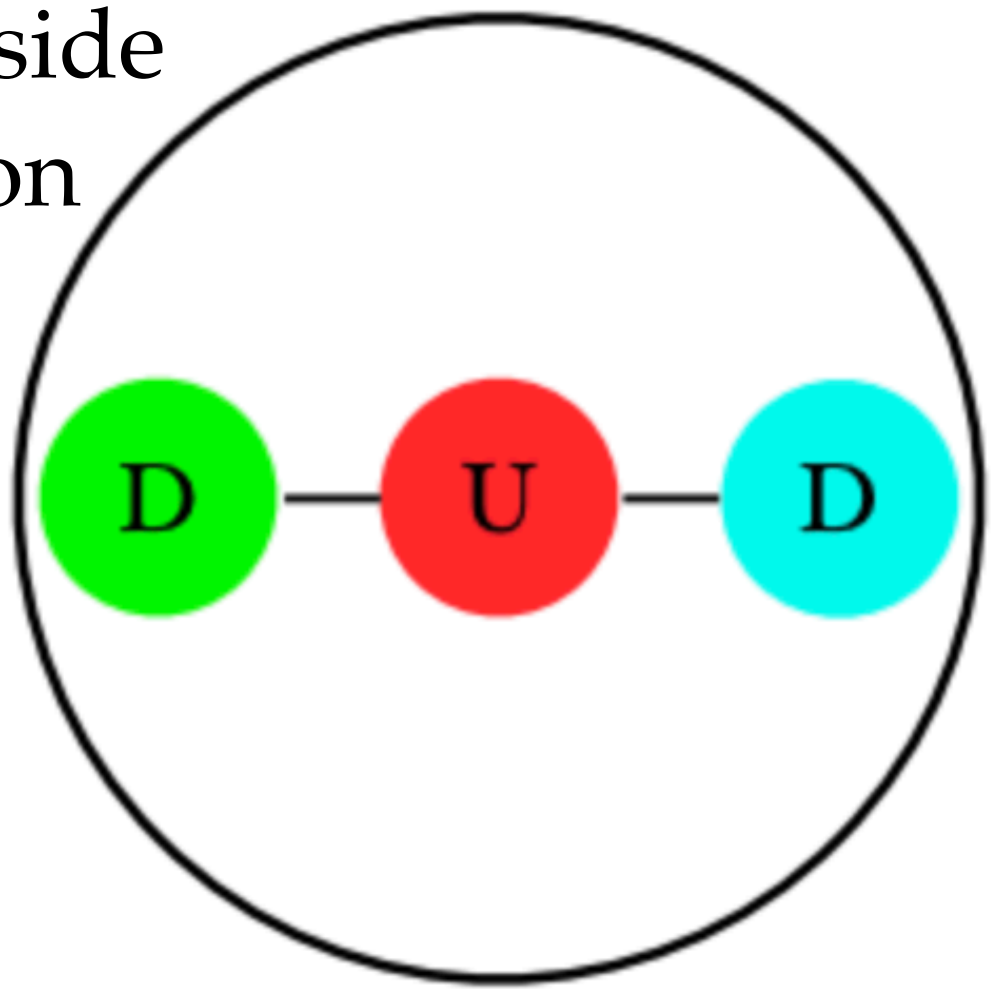
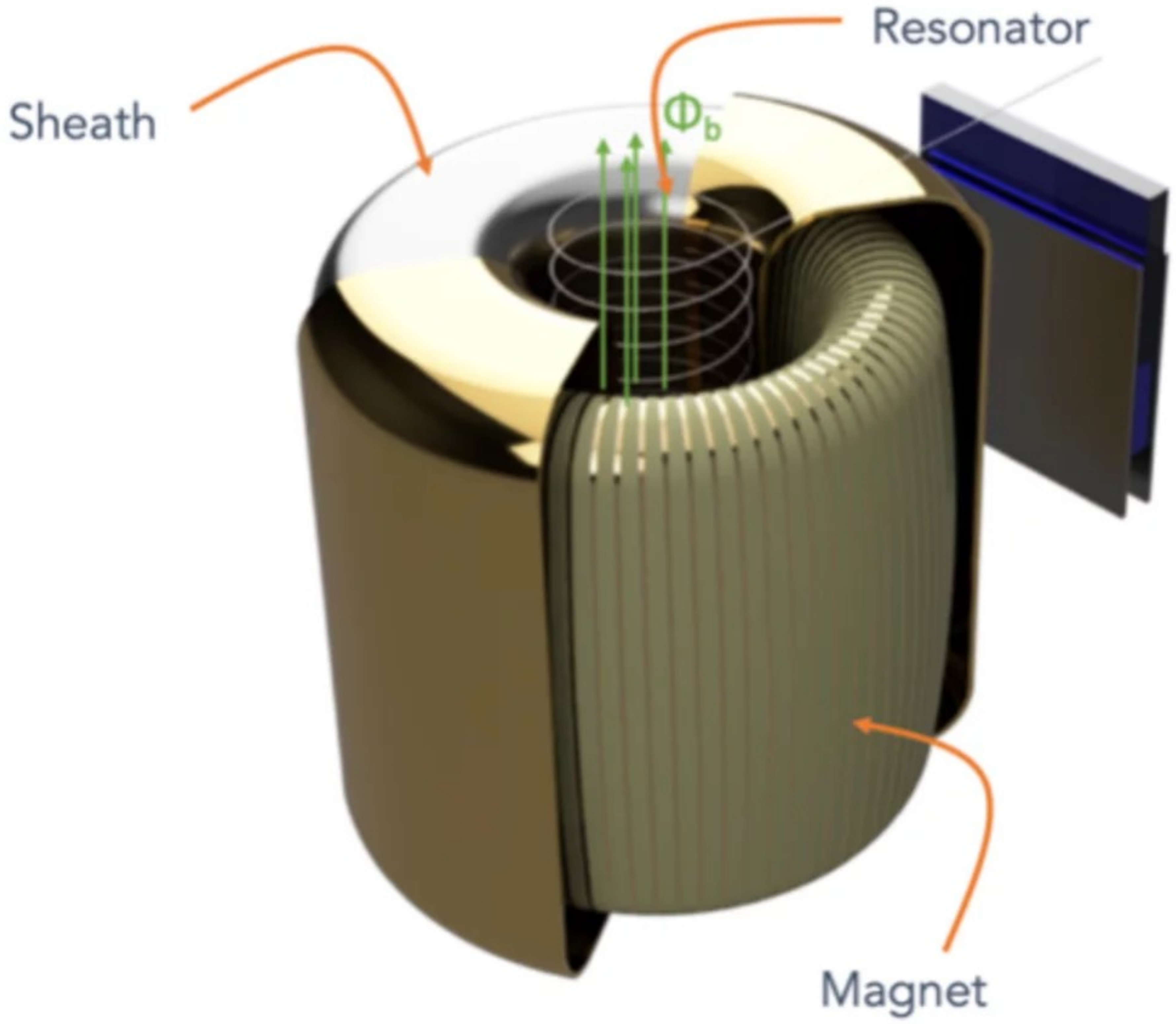


Measured

Supposed to clean up all of our problems …

One promising candidate: The “Axion”

Detection method: Resonance cavity inside a magnetic field



Electric current generated inside the cavity if the size of the cavity matches the wavelength (mass) of the axion

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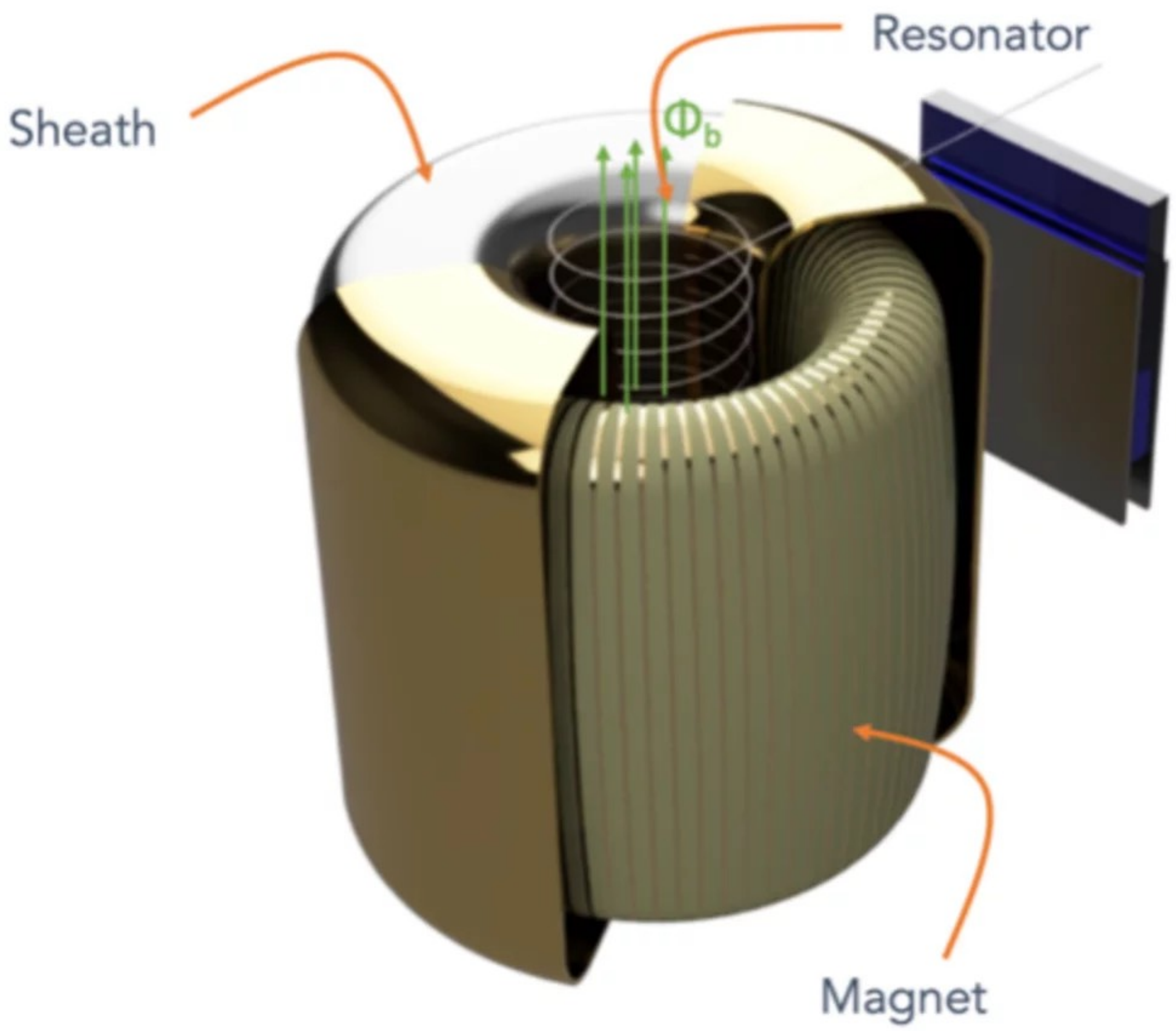

Electric current generated inside the cavity if the size of the cavity matches the wavelength (mass) of the axion

Problem: Mass of the axion is unknown
We don't know what frequency to tune to!

Good news: We can predict the axion mass from theory!

Axion theory has only **one free parameter** f_a: $\mathcal{L}_{PQ} = |\partial\Phi|^2 - \lambda\left(|\Phi|^2 - \frac{f_a^2}{2}\right)^2 - \frac{\lambda T^2}{3}|\Phi|^2$

We know **one measured quantity**: Average dark matter density $\Omega_{DM} = 2.2 \times 10^{-27}\,\mathrm{kg/m^3}$

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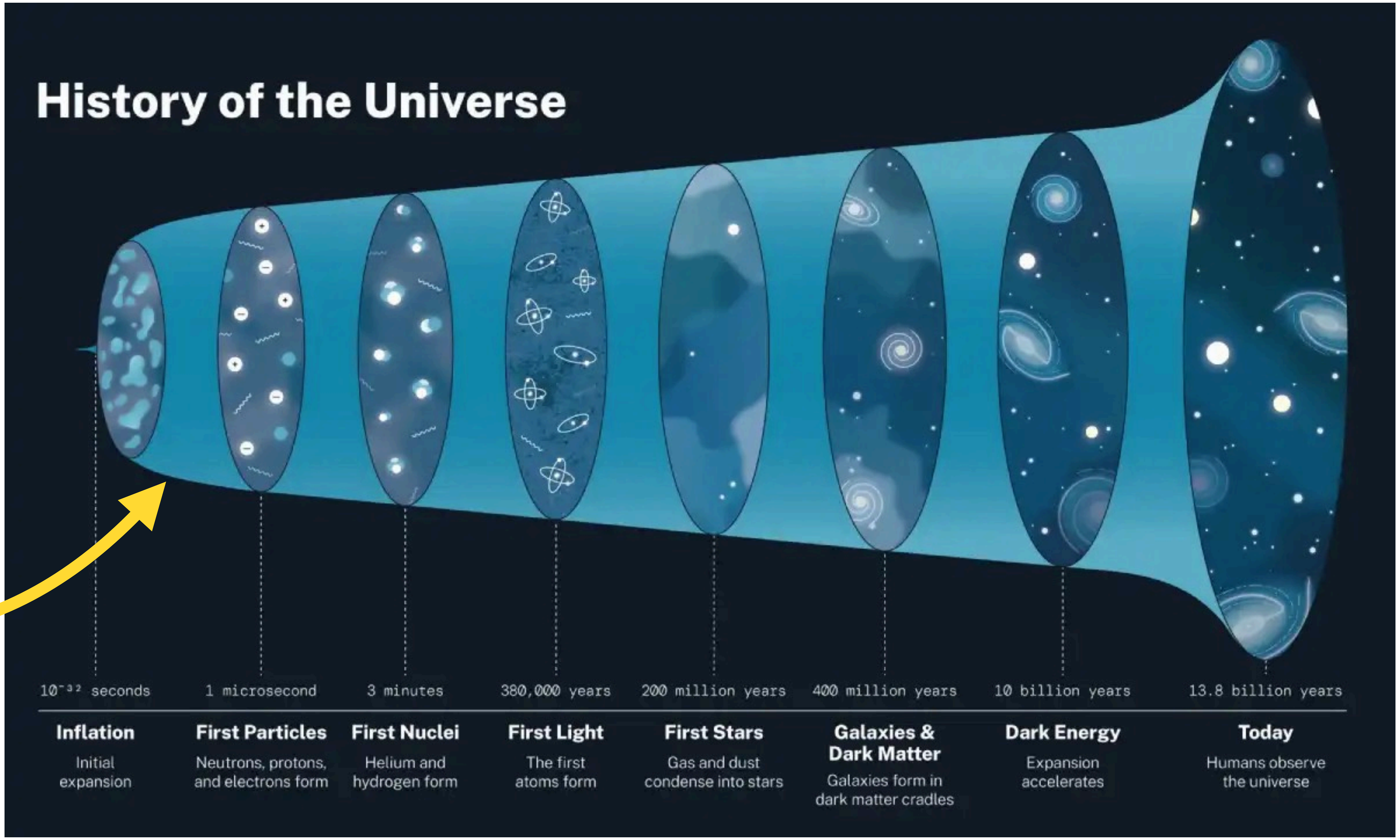
Simulations of the early Universe are needed!



AXIONS

Approach: Solve equations of motions on a 3D lattice in an expanding universe

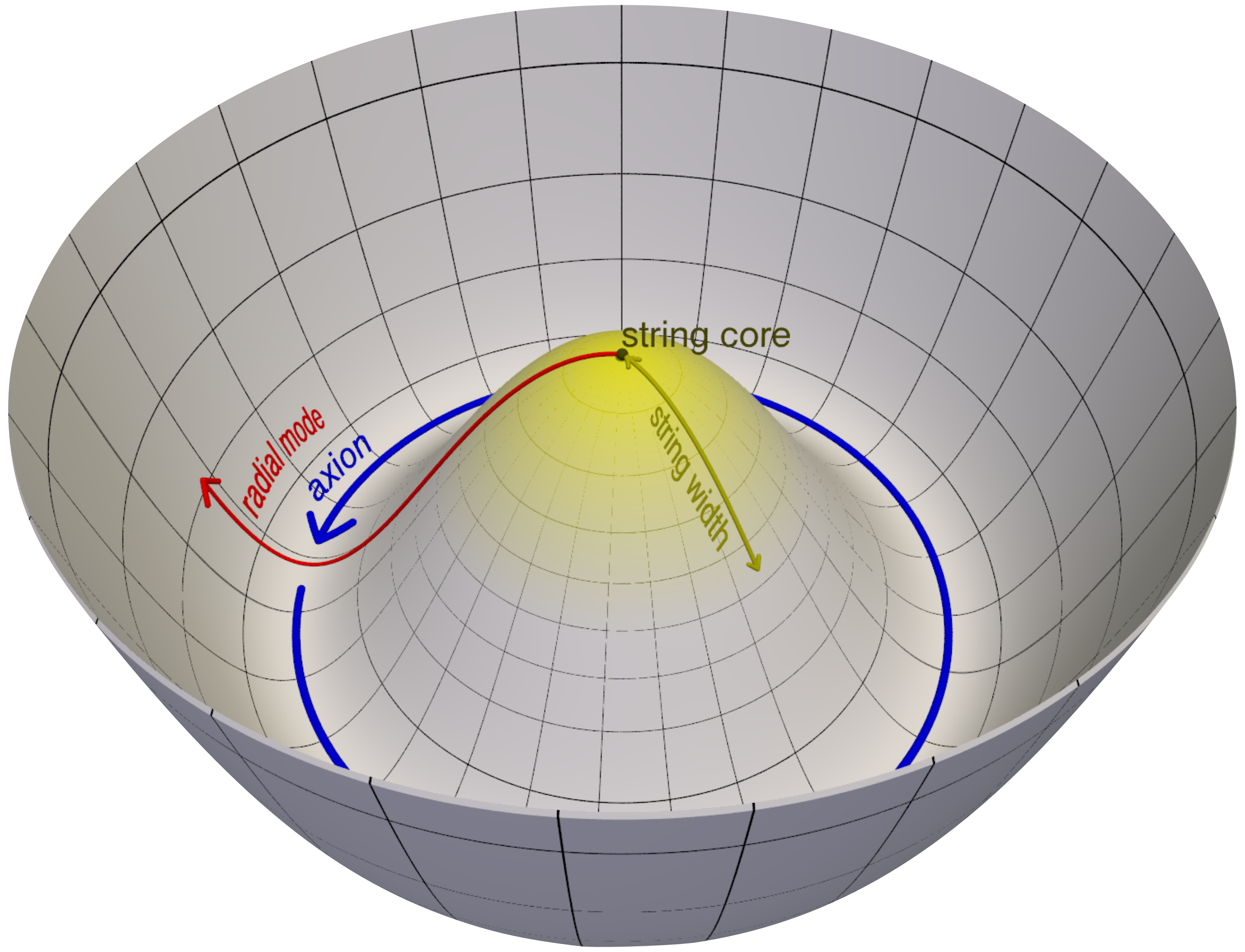
$$\left.\begin{aligned}
\psi_1'' + \frac{2}{\eta}\psi_1' - \bar{\nabla}^2\psi_1 + \lambda\psi_1\left[\eta^2\left(\psi_1^2+\psi_2^2-1\right)+\frac{T_1^2}{3f_a^2}\right] = 0 \\
\psi_2'' + \frac{2}{\eta}\psi_2' - \bar{\nabla}^2\psi_2 + \lambda\psi_2\left[\eta^2\left(\psi_1^2+\psi_2^2-1\right)+\frac{T_1^2}{3f_a^2}\right] = 0
\end{aligned}\right\} \text{Two coupled PDEs on a 3D lattice}$$

Has been done since the 1980s!

AXIONS

Approach: Solve equations of motions on a 3D lattice in an expanding universe

Problem: Potential causes topological defects to form



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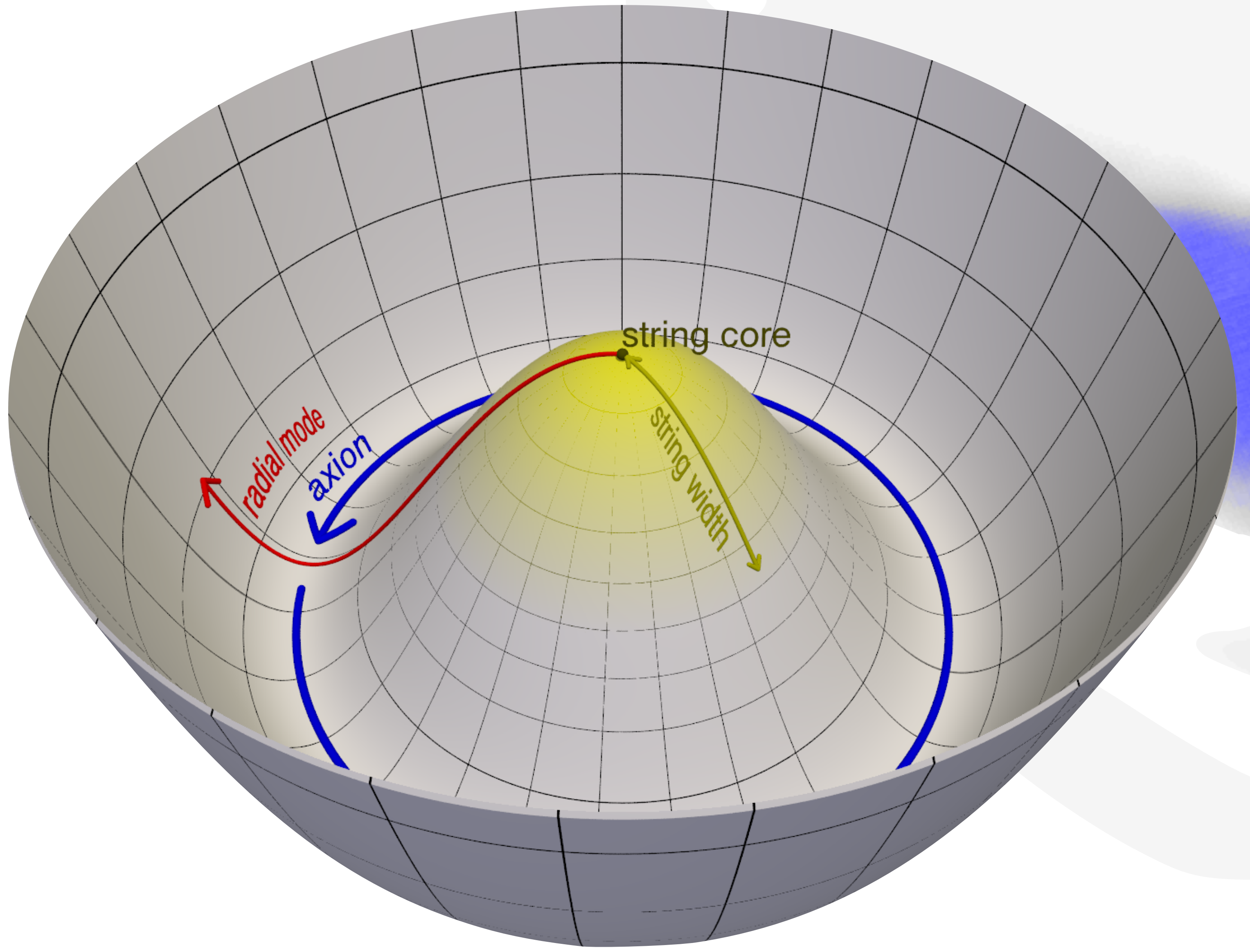

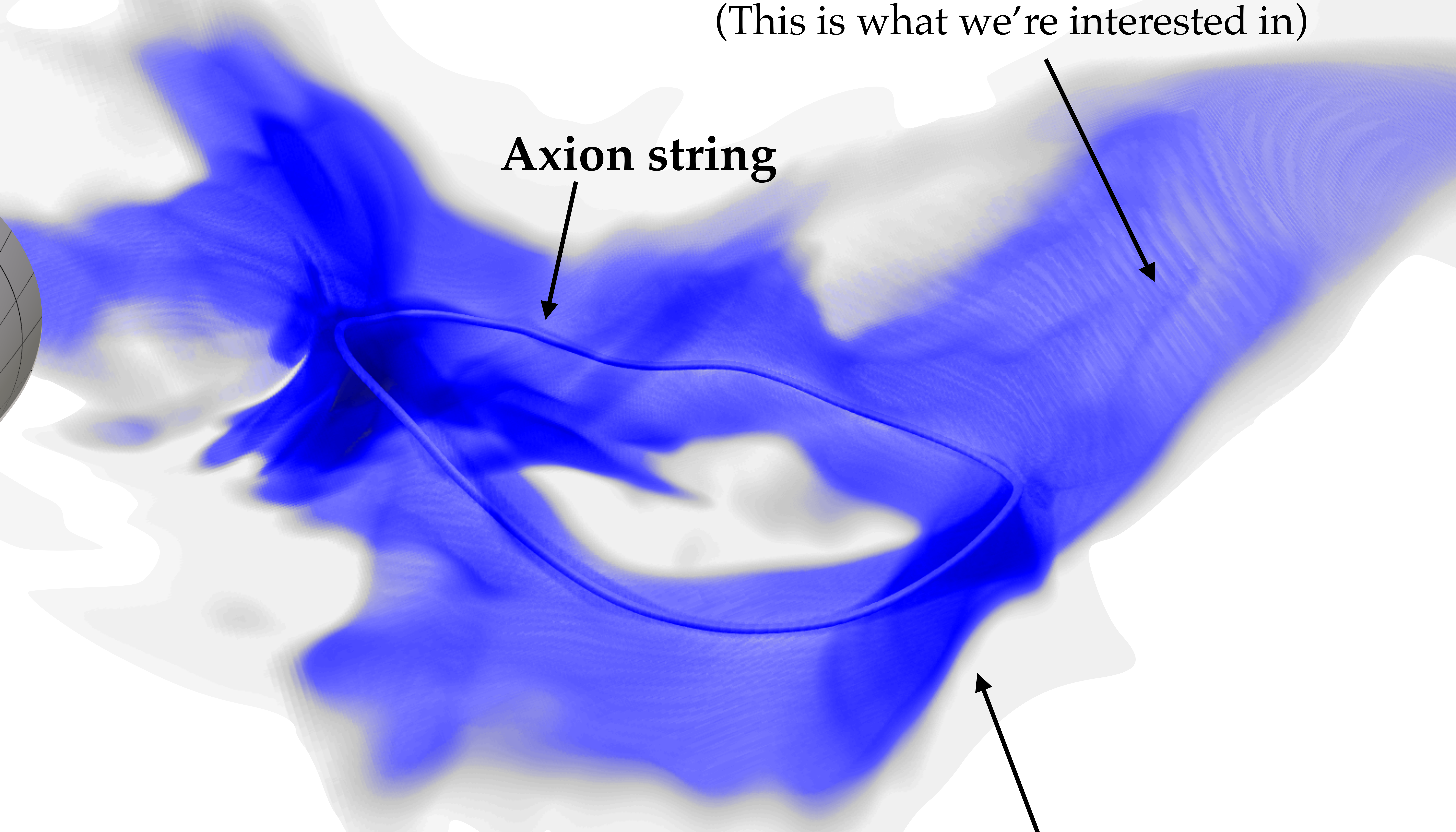

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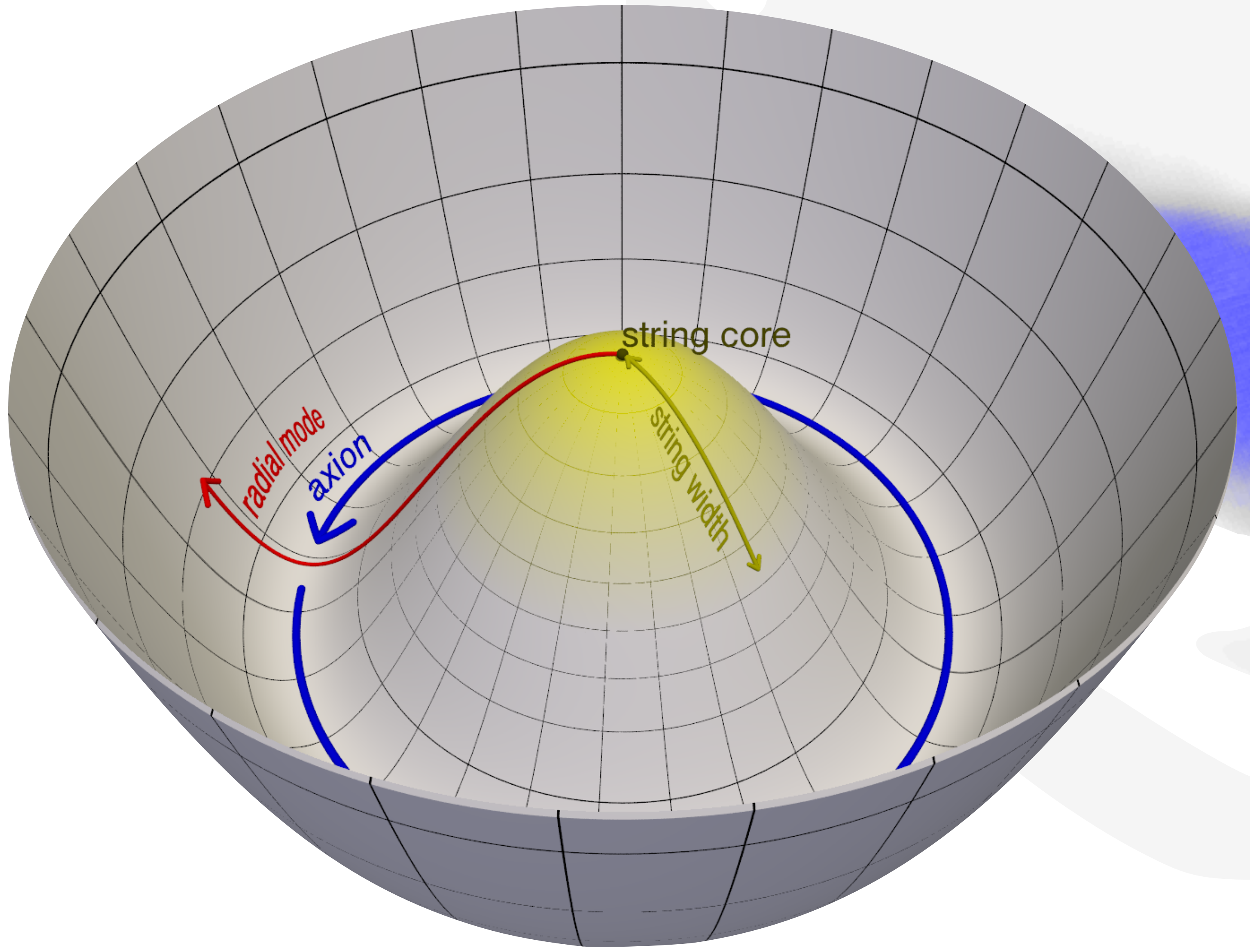
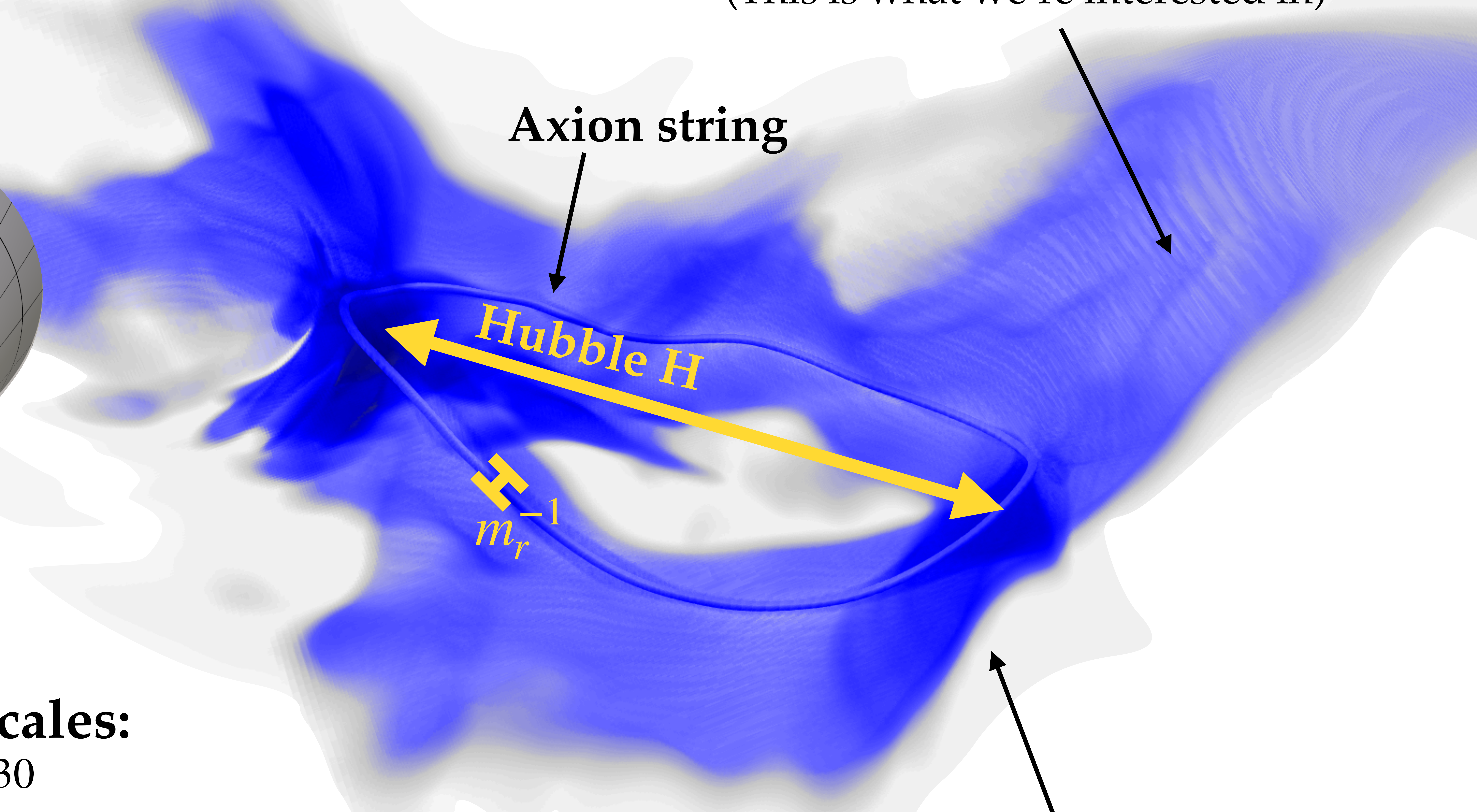




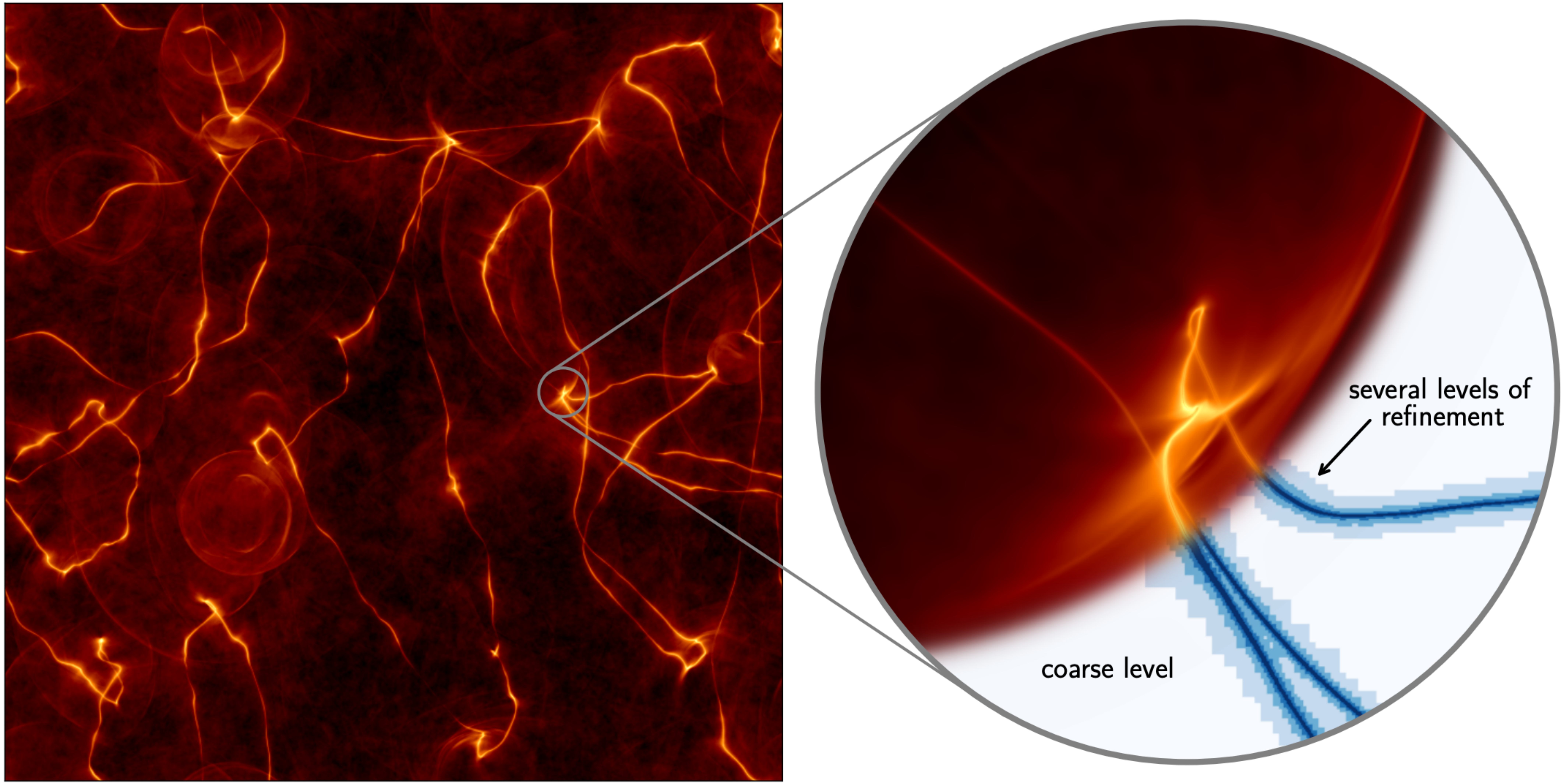
Large separation of scales:

up to $m_r/H \to 10^{30}$

AXIONS

Approach: Solve equations of motions on a 3D lattice in an expanding universe

Perfect use case for AMR!



ADAPTIVE MESH REFINEMENT

AMReX based code:

SLEDGEHAMR:
Sca**L**ar fi**E**ld **D**ynamics **G**etting solv**E**d wit**H** **A**daptive **M**esh **R**efinement

https://github.com/MSABuschmann/sledgehamr
MB, *Astrophys.J.* 979 (2025) 2, 220 • e-Print: 2404.02950

Solves classical equations of motion of coupled scalar fields on GPUs or CPUs

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Solves classical equations of motion of coupled scalar fields on GPUs or CPUs

Many features:

- Several integrators available: Leapfrog, RK2-RK5, low-memory SSPRK3, various Runge-Kutta-Nyström…

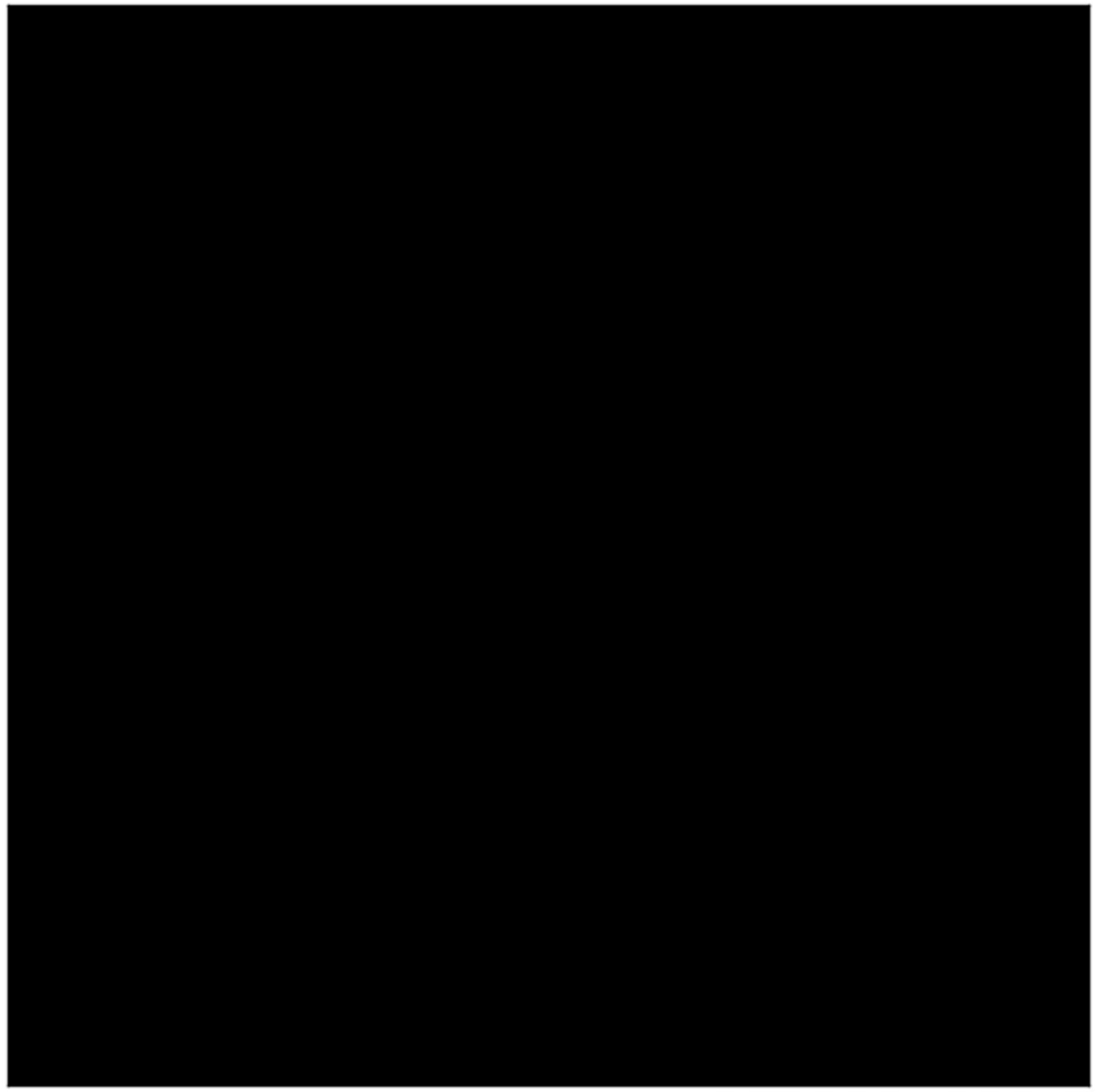
- Many output types such as projections, truncation error estimates, Powerspectra, etc

$$S_X(|\mathbf{k}|) = \frac{Lt}{2\pi N^6} \sum_{\mathbf{k}=|\mathbf{k}|} |\tilde{X}(\mathbf{k})|^2$$

- …

- **Gravitational Wave spectra!**
  The tensor fields are automatically co-simulated as well

$$G(|\mathbf{k}|) = \frac{1}{N^6} \sum_{\mathbf{k}=|\mathbf{k}|} \Lambda_{ij,lm}(\mathbf{k}) \tilde{u}_{ij}(\mathbf{k}) \tilde{u}^*_{lm}(\mathbf{k})$$



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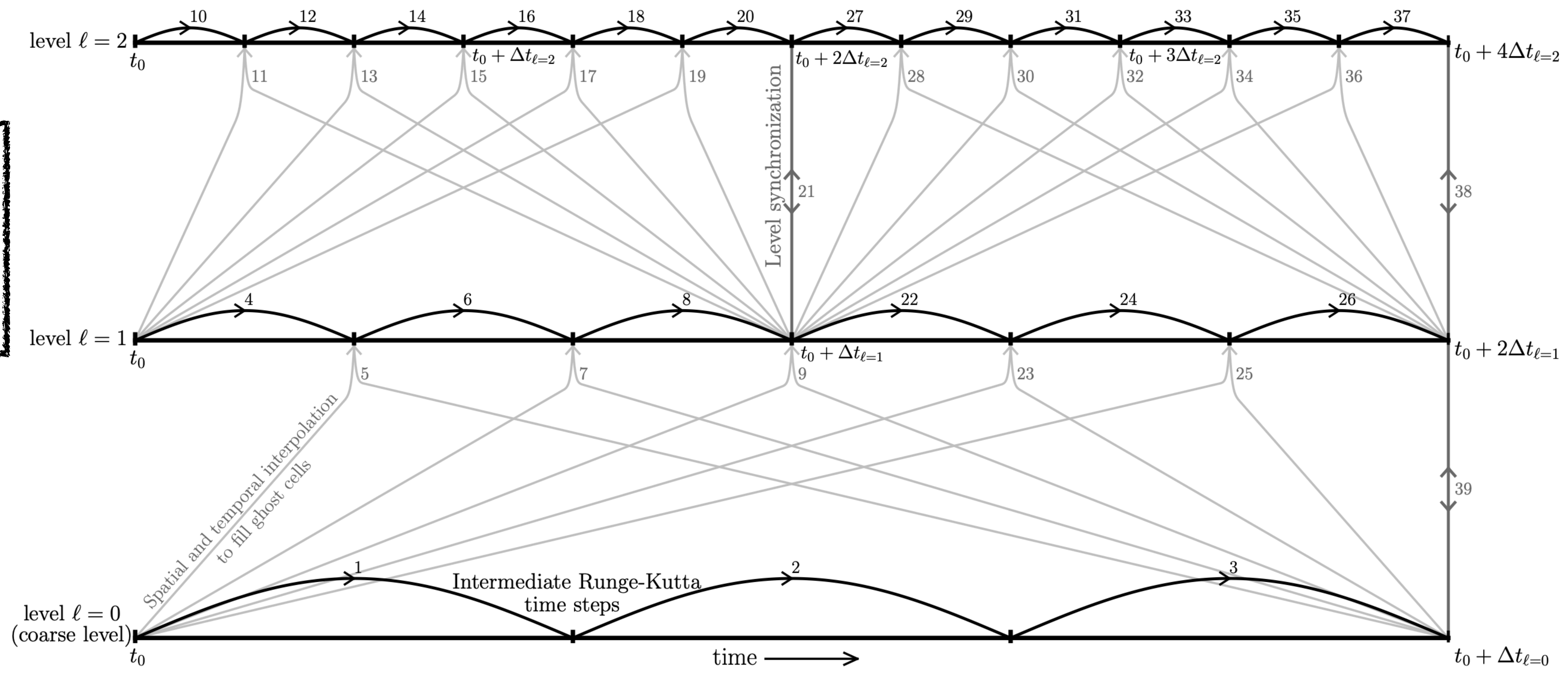

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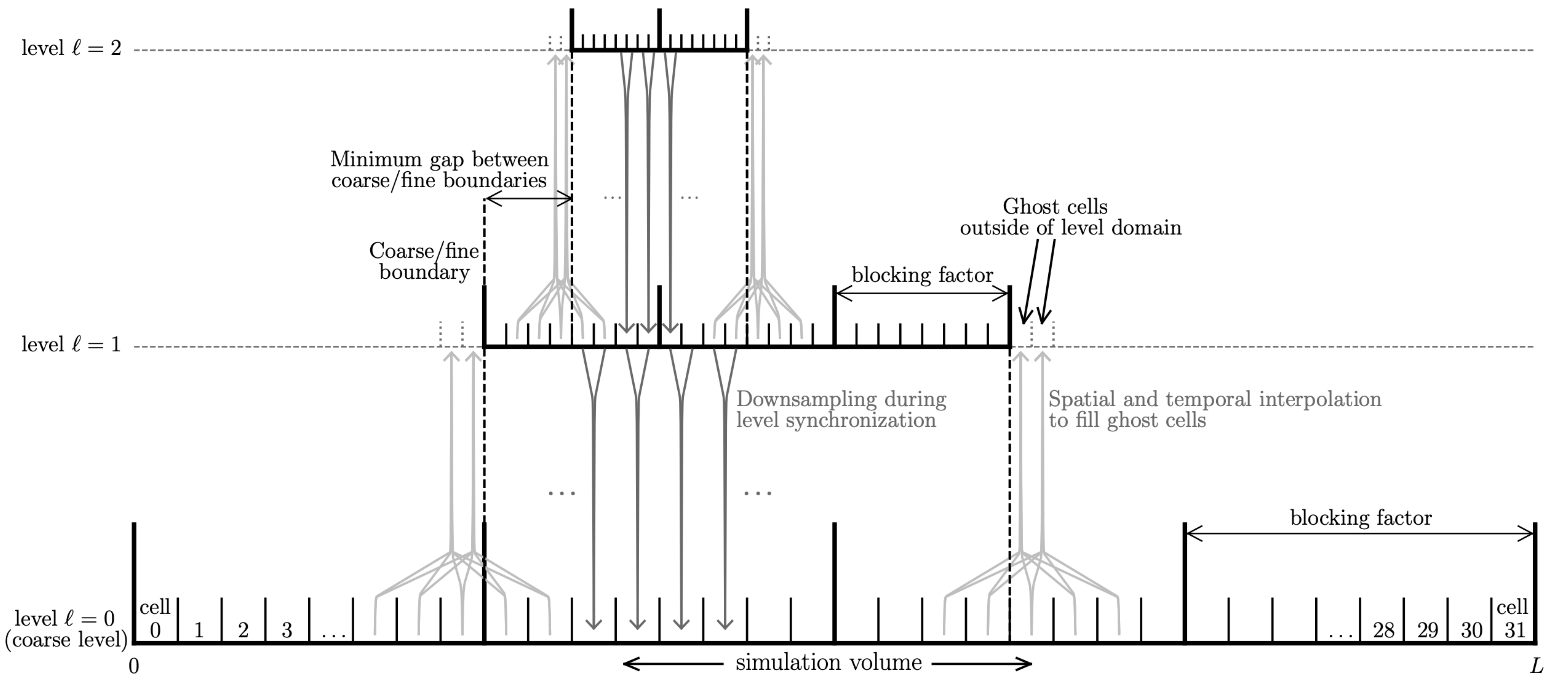
Sca**L**ar fi**E**ld **D**ynamics **G**etting solv**E**d wit**H** **A**daptive **M**esh **R**efinement

Standard evolution techniques:

Timewise: Subsycling-in-time with linear time-interpolation

Spatially: Nested refinement levels with quartic interpolation



ADAPTIVE MESH REFINEMENT

AMReX based code:

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Only equation of motion needed,
AMR will run out of the box
with a default setup that can be tuned.

But, everything is easily customizable by providing hooks to modify the behavior of the code:

Custom cell tagging
Custom diagnostics
Cell injections
…

Modular with limited coding needed:

```
#pragma once
#include <sledgehamr.h>
namespace MinimalExample {

SLEDGEHAMR_ADD_SCALARS(Psi1, Psi2)
SLEDGEHAMR_ADD_CONJUGATE_MOMENTA(Pi1, Pi2)

AMREX_GPU_DEVICE AMREX_FORCE_INLINE
void Rhs(
    const amrex::Array4<double>& rhs,
    const amrex::Array4<const double>& state,
    const int i, const int j, const int k,
    const int lev, const double time,
    const double dt, const double dx,
    const double* params) {

    // Fetch field values.
    double Psi1 = state(i, j, k, Scalar::Psi1);
    double Psi2 = state(i, j, k, Scalar::Psi2);
    double Pi1  = state(i, j, k, Scalar::Pi1);
    double Pi2  = state(i, j, k, Scalar::Pi2);
    double eta = time;

    // Compute Laplacians.
    constexpr int order = 2;
    double laplacian_Psi1 =
        sledgehamr::utils::Laplacian<order>(
            state, i,j,k, Scalar::Psi1, dx*dx);
    double laplacian_Psi2 =
        sledgehamr::utils::Laplacian<order>(
            state, i,j,k, Scalar::Psi2, dx*dx);

    // Compute potential.
    const int a = 0.56233;
    double pot =
        eta*eta*(Psi1*Psi1 + Psi2*Psi2 -1) + a;

    // Now compute the EOMs.
    rhs(i, j, k, Scalar::Psi1) =  Pi1;
    rhs(i, j, k, Scalar::Psi2) =  Pi2;
    rhs(i, j, k, Scalar::Pi1)  =
        -Pi1*2./eta + laplacian_Psi1 - Psi1*pot;
    rhs(i, j, k, Scalar::Pi2)  =
        -Pi2*2./eta + laplacian_Psi2 - Psi2*pot;
}

SLEDGEHAMR_FINISH_SETUP
class MinimalExample
    : public sledgehamr::Sledgehamr {
  public:
   SLEDGEHAMR_INITIALIZE_PROJECT(MinimalExample)
};

}; // namespace
```

ADAPTIVE MESH REFINEMENT

AMReX based code:

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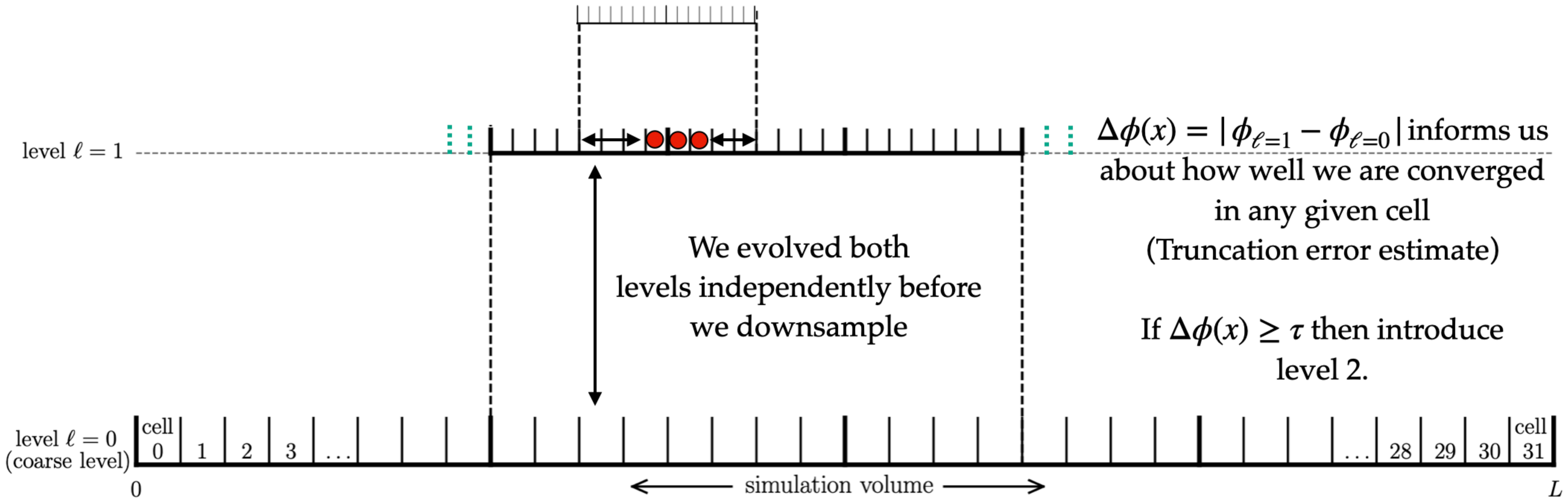
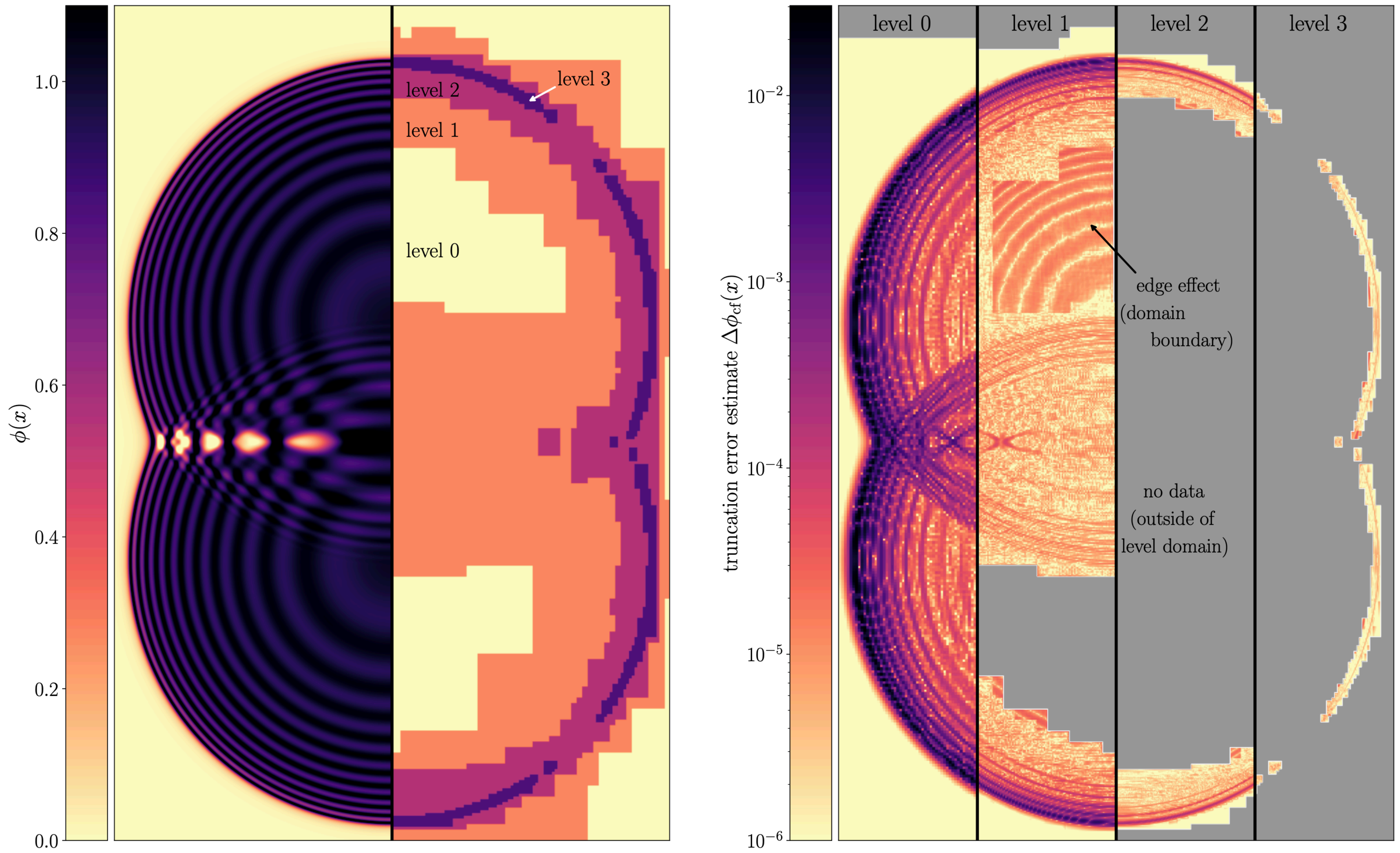


$\Delta\phi(x) = |\phi_{\ell=1} - \phi_{\ell=0}|$ informs us about how well we are converged in any given cell (Truncation error estimate)

If $\Delta\phi(x) \geq \tau$ then introduce level 2.



By default, truncation error estimates are used to judge numerical convergence. Refinement levels are introduced if convergence is bad.

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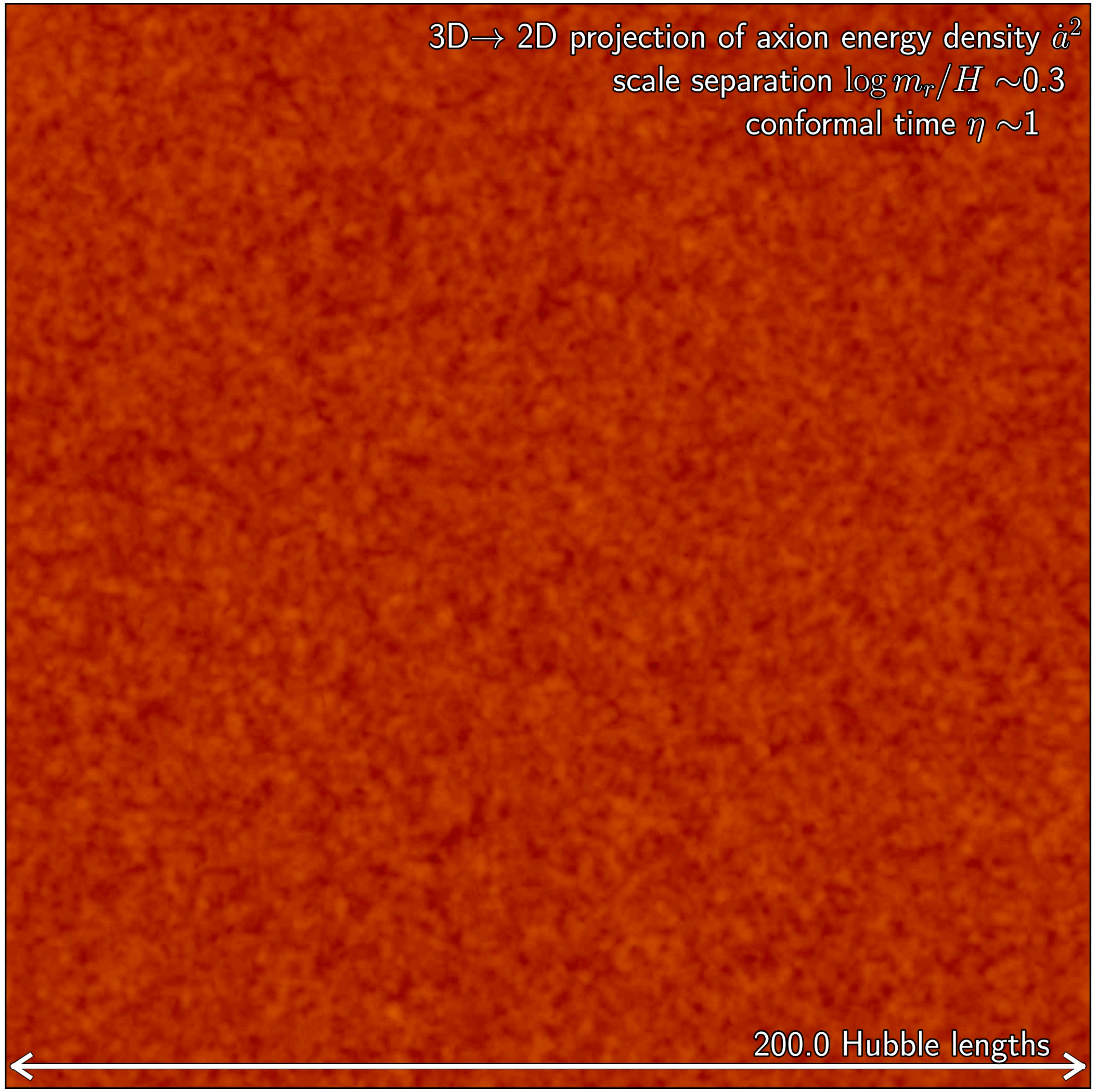

Key data of this simulation

Simulation: 8192^3 + 5 **levels of AMR**
Static lattice would have required 262144^3 cells

Up to **20 Tb per checkpoint**

20+ Million CPU core hours
on Cori & Perlmutter @ NERSC

Increases dynamic range by more than 10^3 over conventional simulations

Result: The most robust axion mass prediction to date!

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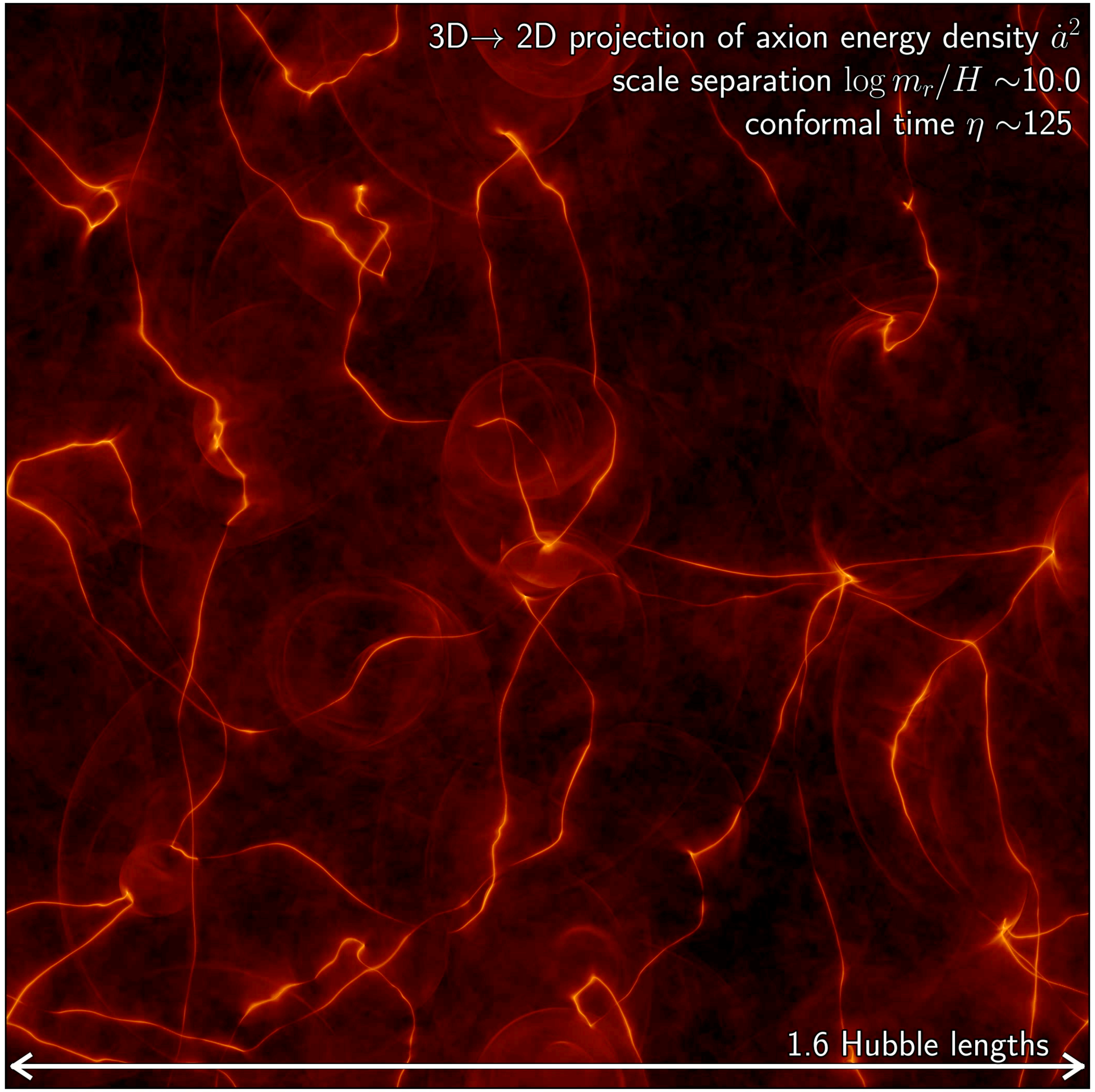

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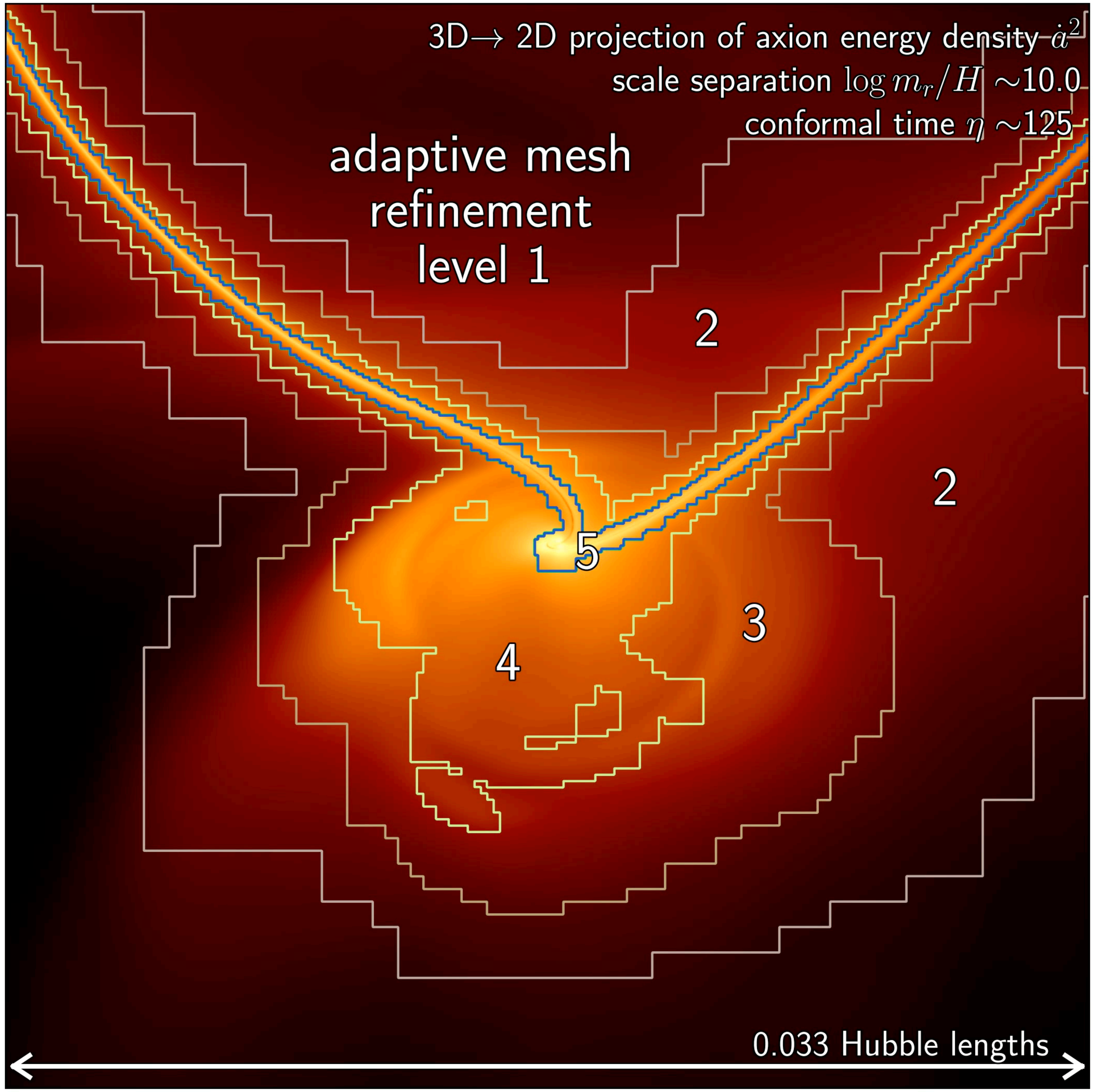

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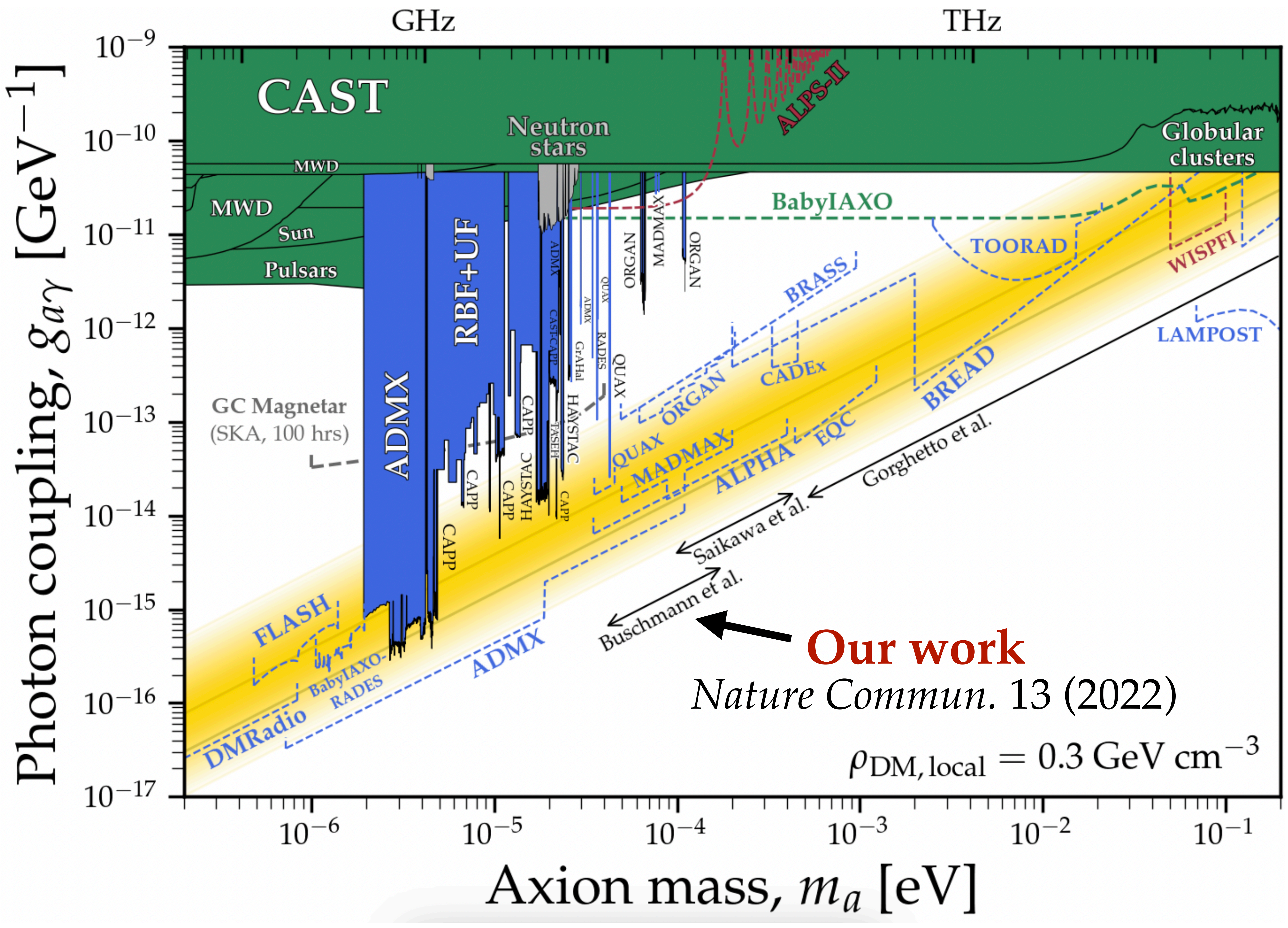

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CHALLENGES

The sheer size of the simulation volume causes challenges:

- **100+ Tb of RAM** required: Cannot run on GPUs
- **Data management.** Do as much analysis on the fly as possible.
- **Bottlenecks**, e.g: regridding (based on Berger-Rigoutsos algorithm)
  Developed alternative algorithm to cut some corners:
  - Pro: 100x faster
  - Con: Less optimal (grid size and load balancing)
    - Grid layout will get worse with repeated application

  Best: Switch between both algorithms dynamically.

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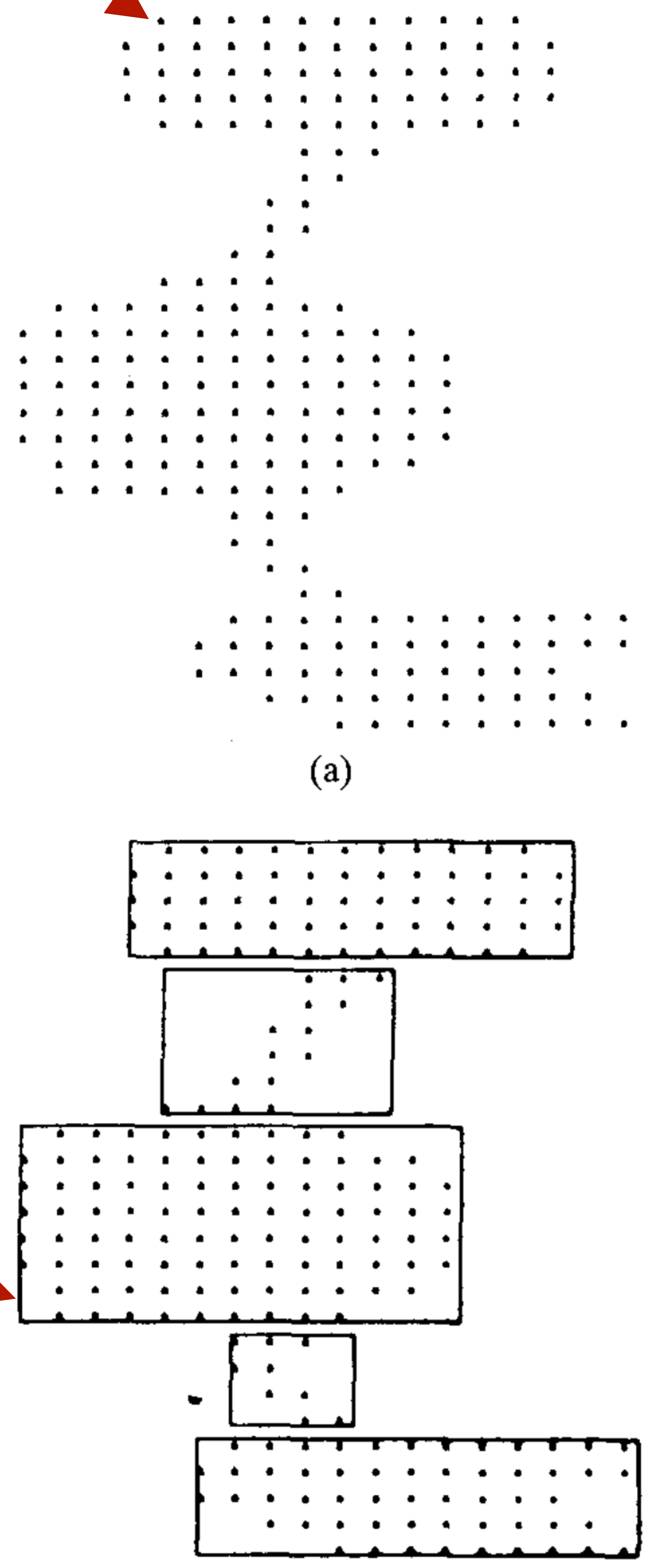

(a)

(b)

We have $N_b > 10^6$ boxes

Clustering scales roughly like $\sim \mathcal{O}(N_t N_b) \to \mathcal{O}(V^2)$

Time-Evolution only $\mathcal{O}(V)$

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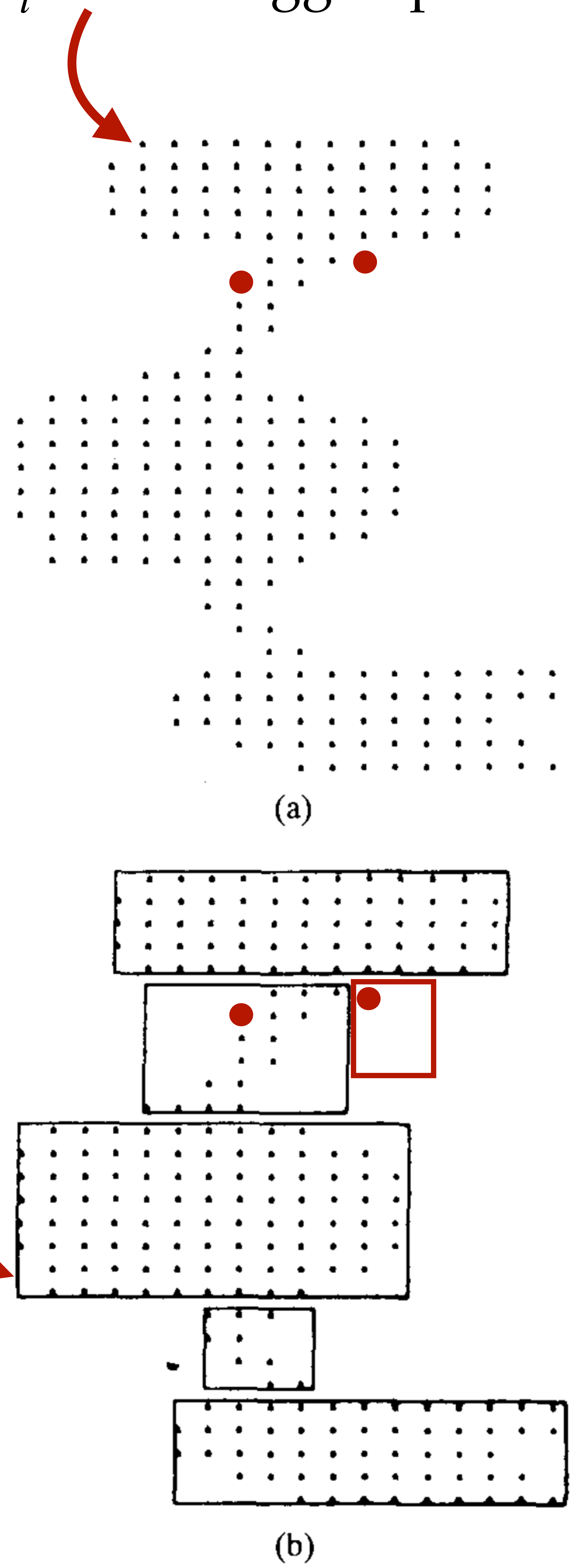
  Con: Less optimal (grid size and load balancing)

  Grid layout will get worse with repeated application

  Best: Switch between both algorithms dynamically.



We have $N_b > 10^6$ boxes

Clustering scales roughly like $\sim \mathcal{O}(N_t N_b) \to \mathcal{O}(V^2)$

Time-Evolution only $\mathcal{O}(V)$

Don't touch existing grid, simply patch locally by adding boxes $\to \mathcal{O}(V)$

THE LIFE AND DEATH OF AXION STRINGS

AMR allows us to simulate the entire life-cycle of the axion string much more accurately
Lots of follow-up projects!